CS 497: Electronic Market Design

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Outline

Auctions

- Methods for allocating goods, tasks, resources,...
- Participants
  - auctioneer
  - bidders
- Enforced agreement between auctioneer and the winning bidder(s)
- Easily implementable (e.g. over the Internet)
- Conventions
  - Auction: one seller and multiple buyers
  - Reverse auction: one buyer and multiple sellers

Auction Settings

- **Private value**: the value of the good depends only on the agent's own preferences
  - e.g a cake that is not resold of showed off
- **Common value:** an agent's value of an item is determined entirely by others' values (valuation of the item is identical for all agents)
  - e.g. treasury bills
- **Correlated value (interdependent value):** agent's value for an item depends partly on its own preferences and partly on others' value for it
  - e.g. auctioning a transportation task when bidders can handle it or reauction it to others

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Four Common Auctions

- English auction
- First-price, sealed-bid auction
- Dutch auction
- Vickrey auction

English auction

aka first-price open-cry auction

- **Protocol:** Each bidder is free to raise their bid. When no bidder is willing to raise, the auction ends and the highest bidder wins. Highest bidder pays its last bid.
- **Strategy:** Series of bids as a function of agent's private value, prior estimates of others' valuations, and past bids
- **Best strategy:**
- **Variations:**
  - Auctioneer controls the rate of increase
  - Open-exit: Bidders have to openly declare exit with no re-entering possibilities

First-price sealed-bid auction

- **Protocol**: Each bidder submits one bid without knowing others' bids. The highest bidder wins the item at the price of it's bid
- **Strategy:** Bid as a function of agent's private value and its prior estimates of others' valuations
- **Best strategy:**

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Example

Assume there are 2 agents (1 and 2) with values v_1, v_2 drawn uniformly from $[0, 1]$. Utility of agent i if it bids b_i and wins is $u_i = v_i - b_i$.

Assume that agent 2's bidding strategy is $b_2(v_2) = v_2/2$. How should 1 bid? (i.e. what is $b(v_1) = z$?).

$$U_1 = \int_{z=0}^{2z} (v_1 - z)dz = (v_1 - z)2z = 2zv_1 - 2z^2$$

Note: given $z = b_2(v_2) = v_2/2$, 1 only wins if $v_2 < 2z$

Therefore,

$$\arg\max_z [2zv_1 - 2z^2] = v_1/2$$

Similar argument for agent 2, assuming $b_1(v_1) = v_1/2$.

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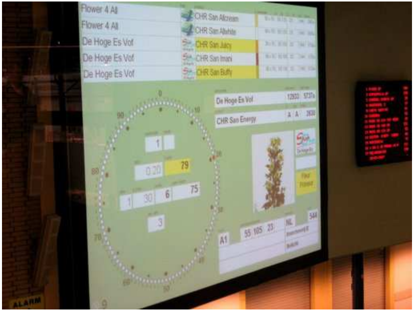
Example

Assume that there are 2 risk-neutral bidders, 1 and 2.

- Agent 1 knows that 2's value is 0 or 100 with equal probability
- 1's value of 400 is common knowledge

What is a Nash equilibrium?

Dutch (Aalsmeer) flower auction


Flower 4 All
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De Hoge Es Vof
De Hoge Es Vof
De Hoge Es Vof
CHR San Juicy
CHR San Imani
CHR San Buffy
De Hoge Es Vof
CHR San Energy
1
0.20
79
1
30
6
75
3
A1
55
105
23
NL
544
ALARM

Dutch auction

Descending auction

- **Protocol:** Auctioneer continuously lowers the price until a bidder takes the item at the current price
- **Strategy:** Bid as a function of agent's private value and prior estimates of others' valuations
- **Best strategy:**
- Dutch flower market, Ontario tobacco auctions, Filene's basement,...

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aka Second price, sealed bid auction

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Vickrey auction

The Vickrey auction is a special case of the Clarke Tax.

- Who pays?
  - The bidder who takes the item away from the others (making the others worse off)
  - Others pay nothing
- How much does the winner pay?
  - The declared value that the good would have had for the others had the winner stayed home (second highest bid)

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Results for Private Value Auctions

- Dutch and first-price sealed-bid auctions are strategically equivalent
- For risk neutral agents, Vickrey and English auctions are strategically equivalent
  - Dominant strategies
- All four auctions allocate item efficiently
  - Assuming no reservation price for the auctioneer

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Revenue

Theorem (Revenue Equivalence)

Suppose that

- *values are independently and identically distributed and*
- *all bidders are risk neutral.*

Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue.

Revenue equivalence fails to hold if agents are not risk neutral.

- Risk averse bidders: Dutch, first-price $\geq$ Vickrey, English
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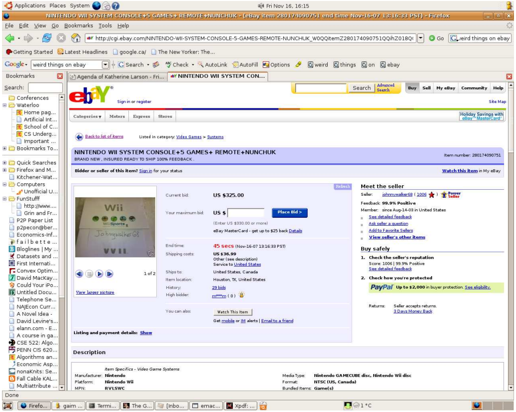
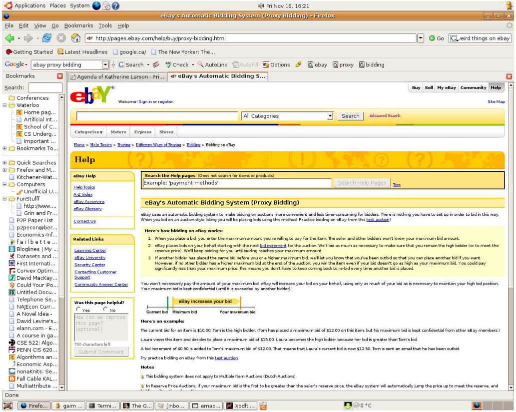
eBay


NINTENDO WII SYSTEM CONSOLE+5 GAMES+ REMOTE+NUNCHUK - (eBay item 280174090751 end time Nov-16-07 13:16:33 PST) - Firefox
NINTENDO WII SYSTEM CONSOLE+5 GAMES+ REMOTE+NUNCHUK
BRAND NEW , INSURED READY TO SHIP 100% FEEDBACK .
Item number: 280174090751
Current bid: US $325.00
Your maximum bid: US $
Place Bid >
End time: 45 secs (Nov-16-07 13:16:33 PST)
Shipping costs: US $36.99
Ships to: United States, Canada
Item location: Houston, TX, United States
History: 29 bids
Meet the seller
Feedback: 99.9% Positive
Buy safely
Description

eBay


eBay's Automatic Bidding System (Proxy Bidding) - Firefox
http://pages.ebay.com/help/buy/proxy-bidding.html
Help
eBay's Automatic Bidding System (Proxy Bidding)
Here's how bidding on eBay works:
eBay increases your bid
Current bid
Minimum bid
Your maximum bid
Here's an example:
Notes

Sniping

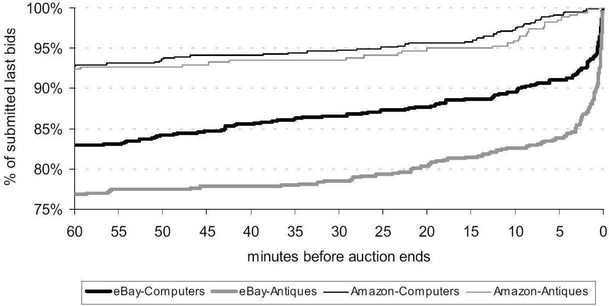


Figure 1a–Cumulative distributions over time of bidders' last bids

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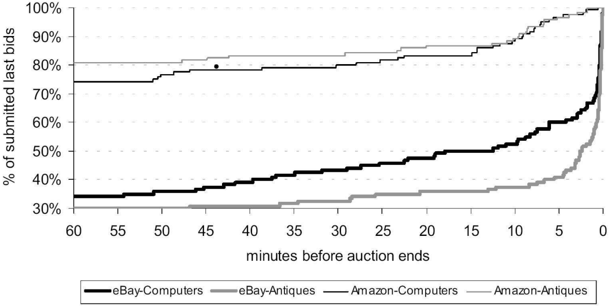
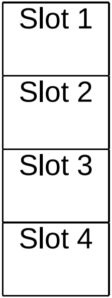


Figure 1b–Cumulative distributions over time of auctions' last bids

Sponsored Search

Sponsored Search



<Keyword>

1. Advertisers are ranked and assigned slots based on the ranking.
2. If an ad is clicked on, only then does the advertiser pay.

Sponsored Search

Rank-by-relevance

- Assign slots in order of (*bid*)(*quality score*)

Bidder	Bid	Quality Score
A	1.50	0.5
B	1.00	0.9
C	0.75	1.5

Ranking
C (1.25)
B (0.9)
A (0.75)

Sponsored Search

- A bidder only pays when its ad is clicked on
- How much does it pay?
  - The lowest price it *could* have bid and still maintained its rank

Sponsored Search

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Ranking
C (1.25)
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C will pay $p = 0.9/1.5 = 0.6$
B will pay $p = 0.75/0.9 = 0.8$
A will pay ?

Sponsored Search

There are many questions about sponsored search

- Is the current way (Generalized Second Price Auction) the best way?
- Revenue?
- Pay-per-what?
- Fraud/vindictive behavior?
- Budgets?
- Should bidders understand how the auction works?
- ...

Selling Multiple Items

So far we have only talked about auctioning a single item. What if we want to sell multiple items?




Multiple Items

- Parallel Auctions
- Sequential Auctions

In both these approaches you have the *exposure problem*.

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Combinatorial Auctions

Allow bidders to submit bids on *bundles* of items.

<(coffee, donut, $5.00)XOR (cake, tea, $4.50)XOR ...>

- Allocation $x^* = \arg\max_x \sum_i^n v_i(x)$ where v_i is the bid of agent i
- Payment $p_i = \sum_{j \neq i} v_j(x') - \sum_{j \neq i} v_j(x^*)$ where x' is the allocation if bidder i had not participated.
- Efficient and truthful!

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Spectrum Auctions

Winner Determination Problem

To run a Combinatorial Auction we must solve

$$x^* = \arg\max_x \sum_i^n v_i(x)$$

- Weighted Set-Packing Problem
- No PTAS

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Winner Determination Problem

- Special structure in the bids
  - Limiting choices for the bidders
- Approximations and heuristics for the WDP
  - Can interfere with the incentive properties of the VCG mechanism
- Throw lots of computing power at the problem

Other issues include

- Communication and preference elicitation
- Design of iterative auctions

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Other research problems

- Computational Limitations and Bidding Behaviour
- Trading Agent Design (Trading Agent Competition)
- Market Design (CATS)
- Trust and Reputation in Online Markets
- Incentive-based computing
- ...