1. Prove from first principles that $\log (n!) \in \Theta(n \log n)$.
2. Prove from first principles that $n \in \omega\left(2^{\sqrt{\log n}}\right)$.
3. Prove or disprove the following claim. If $h_{1}(n) \in \Theta(f(n))$ and $h_{2}(n) \in \Theta(g(n))$, then $\frac{h_{1}(n)}{h_{2}(n)} \in \Theta\left(\frac{f(n)}{g(n)}\right)$. You should prove the statement from first principles or provide a counter example.
4. Provide a tight $\Theta$ bound on the following pseudocode as a function of $n$ :
```
k\leftarrow1
for }i\leftarrow1\mathrm{ to }n\mathrm{ do
    j\leftarrow0
    while }j\leqn\mathrm{ do
        j\leftarrowj+k
    end while
    k\leftarrow2k
end for
```

