# Tutorial 4: October 12

**1.** Consider the problem of sorting an array $A$ of $n$ elements each with multiplicity $n/k$. That is, $A$ consists of $k$ distinct elements $(y_1, y_2, \ldots, y_k)$, where each $y_i$ occurs $n/k$ times in $A$. Prove that any algorithm in the comparison model requires $\Omega(n \log k)$ comparisons to sort $A$ in the worst-case.

Note: $\forall m \geq 1, \left(\frac{m}{e}\right)^m \leq m! \leq m^m$.

**2.** Consider the AVL Tree shown below and perform the following operations: insert 60, delete 72, delete 48.
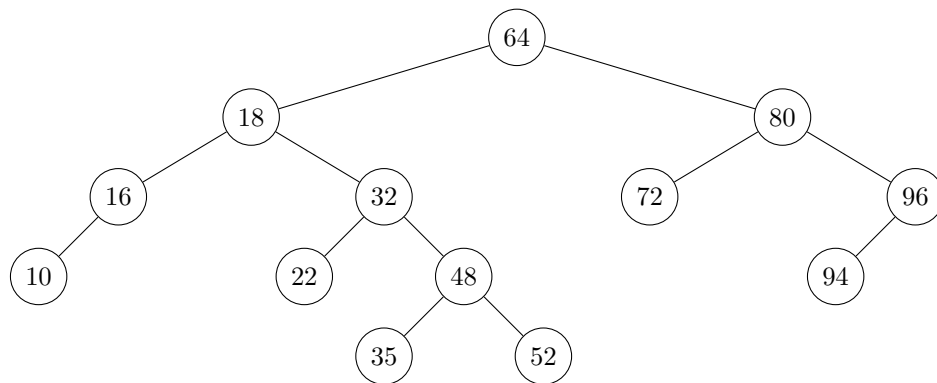


**3.** We consider a modified version of AVL trees where the height difference between the right and left subtrees of any node is in the range $[-2, 2]$ instead of $[-1, 1]$. These are called AVL-2 trees. Prove that the height of an AVL-2 tree on $n$ nodes is in $O(\log n)$.