

## Tutorial 7: November 2

**1.** Consider a hash table of size 7. For each of the scenarios below, insert the keys 14, 10, 20, 13, 7, 17, then delete 14 and search for 13.

- a) Linear Probing with  $h(k) = k \bmod 7$ .
- b) Double Hashing with  $h_1(k) = k \bmod 7$  and  $h_2(k) = (k \bmod 5) + 1$ .
- c) Cuckoo Hashing with  $h_1(k) = k \bmod 7$  and  $h_2(k) = (k \bmod 5) + 1$ .

**2.** Suppose that we use double hashing to resolve collisions, i.e., we use the hash function  $h(k, i) = (h_1(k) + ih_2(k)) \bmod m$ . Show that if  $m$  and  $h_2(k)$  have greatest common divisor  $d \geq 1$  for some key  $k$ , then an unsuccessful insertion for key  $k$  examines  $\frac{1}{d}$ <sup>th</sup> of the hash table before returning to slot  $h_1(k)$ .

Thus, when  $d = 1$ , i.e.,  $m$  and  $h_2(k)$  are relatively prime, then the insertion of  $k$  can only fail if every entry of the hash table is occupied.

**3.** Design a dictionary data structure to store key-value-pairs with uniformly distributed integer keys such that the operations for search, insert, and delete have  $O(\log n)$  runtime and  $O(1)$  expected runtime.