First Example

```
sortednessTester(A, n)
A: array of size n with distinct items
    for i ← 1 to n−1 do
        if A[i−1] > A[i] then return false
    return true
```

Runtime is proportional to the number of comparisons.

$$T^{avg}(n) = \frac{1}{n!} \sum_{\pi \in \Pi_n} T(\pi) = \frac{1}{n!} \sum_{k=1}^{n-1} k \cdot (\#\text{ permutations with exactly k comps})$$

Let π_k be the number of permutations with at least k comparisons.

$$= \frac{1}{n!} \left(\sum_{k=1}^{n-1} k \cdot \pi_k - \sum_{k=1}^{n-1} k \cdot \pi_{k+1} \right)$$

First Example

The number of permutations with at least k comparisons:

- $k = 1$: at least 1 comparison $\Rightarrow$ all $n!$
- $k = 2$: $A[0] < A[1]$ index 0 and 1 occur in sorted order in π
  Example: sorting permutation $\pi = [4, 3, 2, 0, 1]$ or $[4, 0, 3, 2, 1]$, etc.
  $\Rightarrow \binom{n}{2}\ (n-2)!$
- $k = 3$: indices 0, 1, 2 occur in sorted order $\Rightarrow \binom{n}{3}\ (n-3)!$
- k: $0, 1, \ldots, k$ occur in sorted order $\Rightarrow \binom{n}{k}\ (n-k)! = \frac{n!}{k!}$

$$\frac{1}{n!}\left(\sum_{k=1}^{n-1} k \cdot \pi_k - \sum_{k=1}^{n-1} k \cdot \pi_{k+1}\right) = \frac{1}{n!}\left(\sum_{k=1}^{n-1} k \cdot \frac{n!}{k!} - \sum_{k=1}^{n-1} k \cdot \frac{n!}{(k+1)!}\right)$$

First Example

$$
\begin{aligned}
T^{avg}(n) &= \frac{1}{n!}\left(\sum_{k=1}^{n-1} k \cdot \frac{n!}{k!} - \sum_{k=1}^{n-1} k \cdot \frac{n!}{(k+1)!}\right) \\
&= \frac{1}{n!}(\pi_1 + \pi_2 + \pi_3 + \ldots + \pi_{n-1} - (n-1)\pi_n) \\
&= \frac{1}{n!}(\pi_1 + \pi_2 + \pi_3 + \ldots + \pi_{n-1}) \\
&= \frac{1}{n!}\left(\sum_{k=1}^{n-1} \pi_k\right) = \frac{1}{n!}\left(\sum_{k=1}^{n-1} \frac{n!}{k!}\right) \\
&= \sum_{k=1}^{n-1} \frac{1}{k!} < e \approx 2.8 \text{ by Taylor expansion}
\end{aligned}
$$

Average runtime of `sortednessTester` is $O(1)$.
Also, clearly $\Omega(1)$, so, $\Theta(1)$.