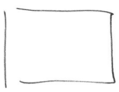
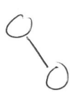
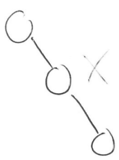
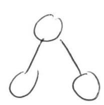
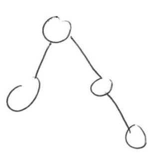
AVL Trees

	height	# nodes
	-1	0
	0	1
	1	2
 		$h=2$ $n=3$
	$h=2$ $n=4$	

Height of an AVL tree

Let $N(h)$ be the least # of nodes in an AVL tree of height h

$$N(h) = \begin{cases} 1 + N(h-1) + N(h-2) & h \geq 1 \\ 1 & h=0 \\ 0 & h=-1 \end{cases}$$

Fibonacci

$F_0 = 0$

$F_1 = 1$

$F_2 = 1$

$F_3 = 2$

$F_4 = 3$

$F_5 = 5$

$N(-1) = 0 = F_2 - 1$

$N(0) = 1 = F_3 - 1$

$N(1) = 2 = F_4 - 1$

$N(2) = 4 = F_5 - 1$

$$N(h) = F_{h+3} - 1$$

$$= \left\lceil \frac{\varphi^{h+3}}{\sqrt{5}} \right\rfloor - 1$$

golden ratio $\varphi = \dfrac{1+\sqrt{5}}{2}$

solve for $h \Rightarrow \sim \log n$

$$N(h) > 2N(h-2)$$
$$> 4N(h-4)$$
$$> 8N(h-6)$$
$$\vdots$$
$$> 2^i N(h-2i)$$
$$\geq 2^{\lfloor h/2 \rfloor} N(0)=1$$

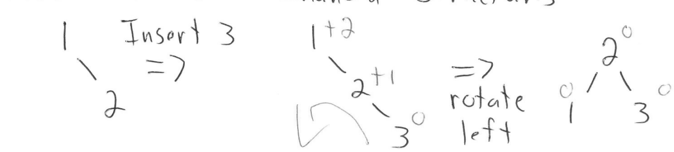
$n > N(h) > 2^{\lfloor h/2 \rfloor}$ solve for h

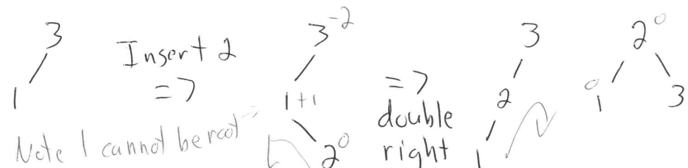
$\Rightarrow h \leq 2 \log n$

An AVL tree with n nodes has height $O(\log n)$

Both heaps & AVL tree have guaranteed height $\Theta(\log n)$. Implementations.

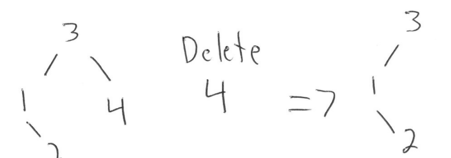
AVL - Basic Unbalanced Structures





Note 1 cannot be root

Note: height of tree before insert is the same after -



Note: height changed

Could be a cascading problem

Delete may require restructure up to root