CS 240: Data Structures and Data Management **Spring 2022**

Tutorial 8: July 4

1. Build a quadtree using the following points: (1, 4), (2, 5), (3, 2), (4, 7), (7, 3), (6, 1), (5, 6), (3, 7).

2. Build a kd-tree using the following points: (1, 4), (2, 5), (3, 2), (4, 7), (7, 3), (6, 1), (5, 6), (3, 7). Note that these are the same points as the previous problem.

3. Suppose that we use double hashing to resolve collisions, i.e., we use the hash function $h(k, i) = (h_1(k) + ih_2(k)) \bmod m$. Show that if m and $h_2(k)$ have greatest common divisor $d \geq 1$ for some key k, then an unsuccessful insertion for key k examines $\frac{1}{d}^{th}$ of the hash table before returning to slot $h_1(k)$.

Thus, when $d = 1$, i.e., m and $h_2(k)$ are relatively prime, then the insertion of k can only fail if every entry of the hash table is occupied.

4. Suppose we have a hash table of size M, where M is prime and strictly greater than 2. Consider a new variation of open addressing called *Quadratic Probing*, with hash function $h(k, i) = (h(k) + i^2) \mod M$. Prove that Quadratic Probing will yield a maximum of only $\lceil \frac{M+1}{2} \rceil$ distinct locations in your hash table for all $i \in \mathbb{Z}^{\geq 0}$