CS 240 – Data Structures and Data Management

Module 10: Compression - Enriched

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Based on lecture notes by many previous cs240 instructors

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Outline

1. Compression
   - Compression ratio
   - Huffman encoding
   - Modified run-length encoding

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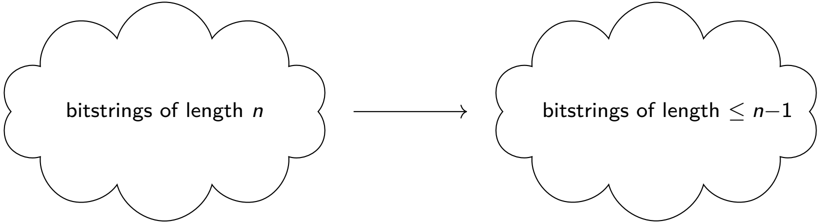
Compression ratio

Theorem: No compression algorithm can have compression ratio < 1 for *all* input strings.

Proof: Assume $\Sigma_S = \Sigma_C = \{0, 1\}$.



- How big are these sets?

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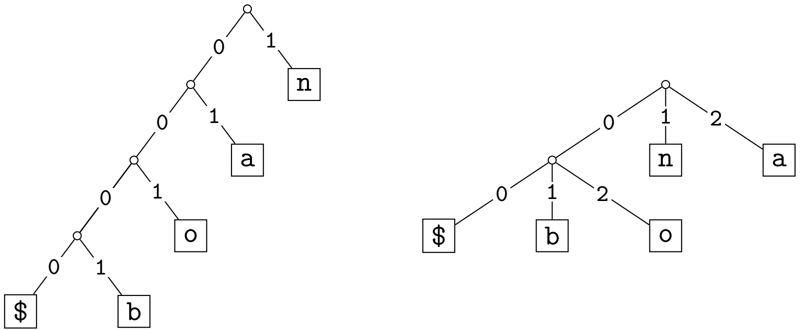
Huffman with a different base

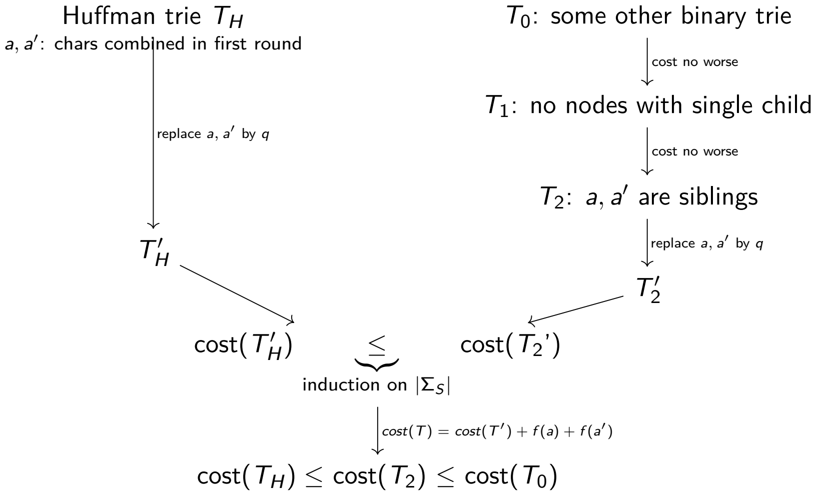
Example text: `nobanana$`, $\Sigma_S = \{\$, b, o, a, n\}$

Character frequencies: `$` : 1,   `b` : 1,   `o` : 1,   `a` : 3,   `n` : 3

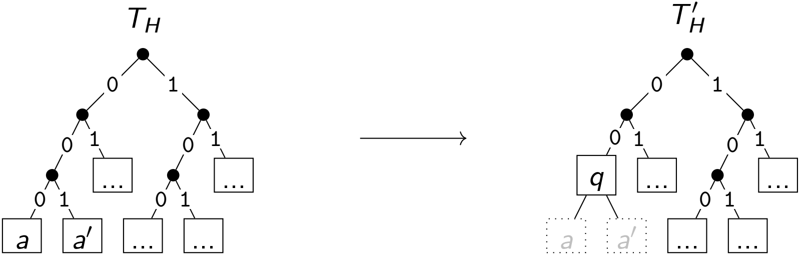


$$\left(\underbrace{10010001011011010000}_{20\ \text{bits}}\right) \text{vs.} (202011212100)_3 = \left(\underbrace{1100000111110010110}_{19\ \text{bits}}\right)$$

Huffman optimality – outline



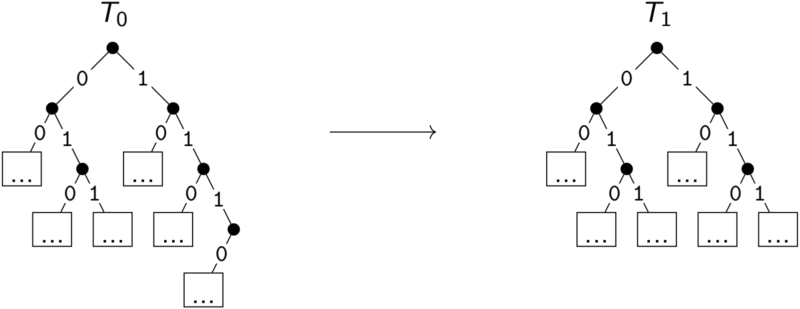
Huffman optimality – from T_H to T'_H



Observation: $cost(T_H) = cost(T'_H) + f(a) + f(a')$

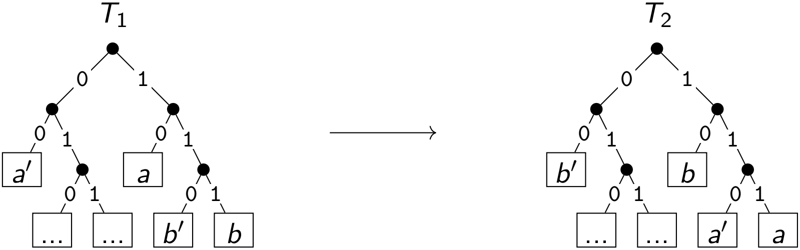
Observation: T'_H is the Huffman-encoding trie for $\Sigma_S \setminus \{a, a'\} \cup \{q\}$

Huffman optimality – from T_0 to T_1



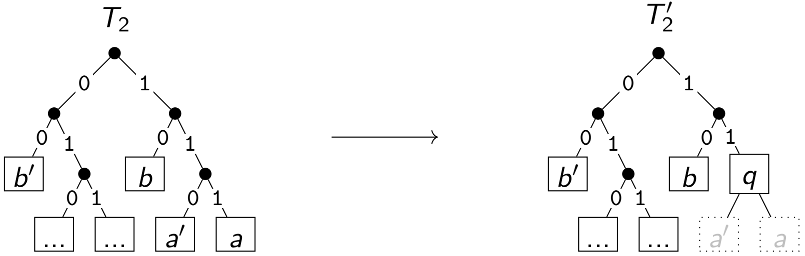
Observation: $cost(T_1) \leq cost(T_0)$

Huffman optimality – from T_1 to T_2



Observation: $cost(T_2) \leq cost(T_1)$

Huffman optimality – from T_2 to T_2'



Observation: $cost(T_2) = cost(T_2') + f(a) + f(a')$

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Modified run-length encoding

Binary bijective numeration (use here $\{A', B'\}$ rather than $\{0, 1\}$):

k	1	2	3	4	5	6	7	8	...
$E(k)$	A'	B'	$A'A'$	$A'B'$	$B'A'$	$B'B'$	$A'A'A'$	$A'A'B'$	...

- $E(k)$ has length $\lfloor \log(k+1) \rfloor$
- Base-2 representation has length $\lfloor \log k \rfloor + 1$, so this is (slightly) shorter.

Modified RLE: Encode only runs of 0, using binary bijection numeration:

Example:

$$S = 110, 114, 100, \underbrace{0, 0, 0, 0}_{4}, 1, 6, 100, 2, \underbrace{0, 0}_{2}, 103, 2, \underbrace{0, 0, 0, 0}_{4}$$

$$C = 110, 114, 100, A', B', 1, 6, 100, 2, B', 103, 2, A', B'$$