CS 240E: Data Structures and Data Management **Winter 2022**

Final Help Session

Note: This is a sample of problems designed to help prepare for the final exam. These problems do *not* encompass the entire coverage of the exam, and should not be used as a reference for its content. Also, these problems aren't organized by difficulty, but by the order in which the relevant concepts were taught.

1 True/False

For each statement below, write true or false.

a) Open addressing hashing that uses linear probing will require two hash functions.

b) Run-length encoding may result in text expansion on some strings.

c) When doing range search on a quadtree, if there is no point within the range specified, the worst case runtime complexity is in $\Theta(h)$.

d) Suffix trees for pattern matching require preprocessing the pattern.

e) Inserting a set of keys into an empty compressed trie will always result in the same final trie regardless of the insertion order.

f) The runtime complexity of range query for kd-trees depends on the spread factor of points. The spread factor is the ratio of the side length of the minimum bounding box (whose bottom left corner is at (0,0)) to to the minimum distance between the points. We assume the points have nonnegative coordinates (we can translate them if necessary).

g) When using KMP to search for the pattern $\texttt{a}^m$ in the text $\texttt{a}^{n-1}b$, the positions of the pattern shifts are the same as the brute-force algorithm.

h) Rehashing may be required in Cuckoo Hashing even if the load factor is at an acceptable value.

i) Every AVL Tree is also a 2-4 Tree.

j) Move-to-front transformation uses adaptive instead of static dictionaries.

2 Multiple Choice

Pick the best answer for each question.

a) Which of the following functions $f(i)$ would cause interpolation search to have the least worst-case runtime on an array A with $A[i] = f(i)$?

   a) $f(i) = \log(i)$

b) $f(i) = i$
c) $f(i) = i^2$
d) $f(i) = 2^i$

b) Given $h_0(k) = k \bmod 7$ with two hash tables, each of size 7, which of the following hash functions would be most suitable for h_1 in double hashing?

i) $h_1(k) = k^2 \bmod 7$
ii) $h_1(k) = (k \bmod 6) + 1$
iii) $h_1(k) = 2 \cdot (k \bmod 4)$
iv) $h_1(k) = \left\lfloor \frac{1}{2} \cdot (k \bmod 13) \right\rfloor$

c) Given $h_0(k) = k \bmod 7$ with two hash tables, each of size 7, which of the following hash functions would be most suitable for h_1 in cuckoo hashing?

i) $h_1(k) = k^2 \bmod 7$
ii) $h_1(k) = (k \bmod 6) + 1$
iii) $h_1(k) = 2 \cdot (k \bmod 4)$
iv) $h_1(k) = \left\lfloor \frac{1}{2} \cdot (k \bmod 13) \right\rfloor$

d) If the root of a quadtree represents the region $[0, 128) \times [0, 128)$ while the deepest (lowest) internal node represents the region $[88, 92) \times [24, 28)$, what is the height of the quadtree?

a) 4
b) 5
c) 6
d) 7

e) Which one of the following statements about compressed tries is false?

i) Every internal node stores an index indicating the bit position to be tested on a search.
ii) The root of the compressed trie always tests the first bit.
iii) A compressed trie that stores n keys always contains less than n internal nodes.
iv) The height of a compressed trie never exceeds the length of the longest string it stores.

f) Which of the following search operations on a non-dictionary structure has the most efficient worst-case runtime?

i) Searching for a specific key in a max-heap.
ii) Searching for a specific point in a kd-tree with points in general position.
iii) Searching for any occurrence of a specific character in a text using a suffix tree, with children pointers stored as arrays.
iv) Searching for a specific character in a decoding trie of characters (like Huffman's Trie)

3 Hashing

Let $p \geq 3$ be prime, and consider the universe of keys $U = \{0, 1, \ldots, p^2 - 1\}$. Answer each question for an initially empty hash table of size p.

a) Using double hashing with $h_1(k) = k \bmod p$ and $h_2(k) = \lfloor k/p \rfloor + 1$, give a sequence of **two** keys to be inserted that results in failure.

b) Using cuckoo hashing with $h_1(k) = k \bmod p$ and $h_2(k) = k \bmod (p-1) + 1$ and **one** table, give a sequence of **three** keys to be inserted that results in failure.

c) Using cuckoo hashing with $h_1(k) = k \bmod p$ and $h_2(k) = \lfloor k/p \rfloor$ and **one** table, give a sequence of **three** keys to be inserted that results in failure.

4 Boyer-Moore

Boyer-Moore can be modified in many ways. For each of the modifications listed below, state whether or not the modification is valid, i.e., the modified Boyer-Moore will always successfully find the first occurrence of P in T, if P appears in T, or return FAIL if P is not in T. If the answer is "Yes", provide a brief explanation of why it is still valid. If the answer is "No", demonstrate a counterexample, i.e., trace the algorithm on a specific P and T of your choice where the result is incorrect.

Note: We are using Boyer-Moore with only the Bad Character heuristic here.

a) Using a first-occurrence function (denoting the index of the first occurrence of the argument character) instead of a last-occurrence function.

b) When checking a pattern shift, compare characters from the start of the pattern and move forward, instead of scanning backwards from the end of the pattern.

c) Use the last-occurrence function for $P[0 \ldots m-2]$, i.e., P with its last character removed, instead of the last-occurrence function for P.

5 Quad Trees

a) Create a set of 8 distinct points for which all coordinates are integers in the range $[0, 8)$ and that has the following quad-tree.

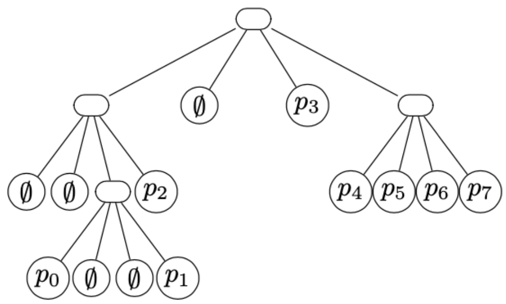
b) Given a quad-tree T, what is the smallest integer k such that there exists a set of distinct points whose quad-tree is T and whose coordinates are integers in the range $[0, 2^k)$?

6 Range Queries

Consider the following set of points in $[0, 16] \times [0, 16]$:

$p_0 : (3, 5), p_1 : (7, 8), p_2 : (6, 2), p_3 : (8, 0), p_4 : (0, 3), p_5 : (4, 6), p_6 : (2, 9), p_7 : (9, 1)$

a) Show the corresponding quad-tree.



b) Show the corresponding kd-tree.

c) Show one possible range tree. The primary tree should be perfectly balanced.

d) Show the corresponding Cartesian tree.

e) Show the priority search tree where the split-line coordinate is always the upper median.

7 Pattern Matching

Consider the pattern $P = 0110101$ and the text T listed in the following table.

0	0	1	1	1	1	0	0	1	1	1

a) Indicate all the checks that were done by the brute-force method.

b) Consider the Karp-Rabin fingerprints that simply counts the number of 1s in the bit-string. Is this a rolling hash-function? And using these fingerprints, how many checks where done during Karp-Rabin pattern matching?

c) Compute the KMP failure-function for P.

d) Show the KMP-automaton for P.

e) Show the good-suffix array (from the Boyer-Moore heuristic) for P.

f) Consider now the pattern P=fiddledidi. Show the last-occurrence array.

8 Suffix Trees

Jason discovered a secret message in the form of a Suffix Tree S, indicating the location of a hidden treasure.

a) Design an algorithm that recovers the original text T from its corresponding suffix tree S. The algorithm should run in $O(n)$ time while using $O(n)$ auxiliary space.

b) Determine the original text for the following suffix tree:

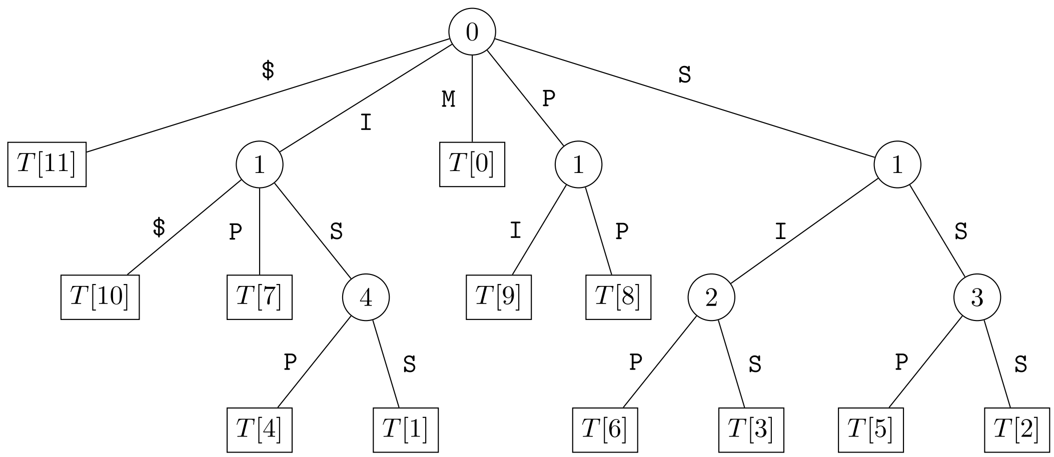


Figure 1: Mysterious Suffix Tree

9 Move-to-Front + Run-Length Encoding

Consider an ecoding algorithm tht utilizes the following fixed dictionary, where the alphabet consists of letters from A to P:

Char	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Code	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The steps of the encoding algorithm are:

- Encode each character with the dictionary above using 4-bit codewords, while also applying Move-To-Front.
- Encoding the resulting string with Run-Length-Encoding.

a) Decode the string 1000101100110011, which was encoded using the algorithm described.

b) For each $n > 1$, give an example of a valid string whose encoding has the minimum number of bits over all strings of length n.

c) For each $n > 1$, give an example of a valid string whose encoding has the maximum number of bits over all strings of length n.

10 Consecutive Trie Strings

Given an uncompressed trie T that stores a list of binary strings, design an algorithm $Consecutive(b_1, b_2)$ that takes two binary strings in T as input, and outputs true if the strings are consecutive in pre-order traversal of the trie, and outputs false otherwise. Assume that branches are ordered as $, 0, 1. The runtime should be bounded by $O(|b_1| + |b_2|)$.

For example, suppose T stores $\{000, 01, 0110, 101, 11\}$.

$Consecutive(0110, 101)$ outputs true.
$Consecutive(01, 000)$ outputs true.
$Consecutive(11, 000)$ outputs false.

11 B-Trees

Consider the following B-Tree, of order 5:

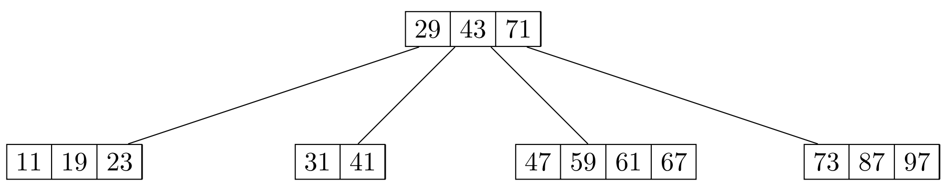


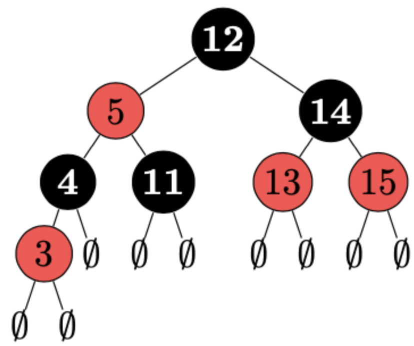
Figure 2: B-Tree of order 5

a) Insert the following keys into the B-Tree, in the order given: 13, 53, 17. Show the tree after each insertion.

b) Delete the following keys from the original B-Tree, in the order given: 19, 43, 31, 29. When deciding between successor/predecessor, choose the successor. When deciding between left or right sibling for transfer/merge, select the right sibling. Show the tree after each deletion.

12 Red-black Trees

Consider the following red-black tree.



a) Show the 2-4 tree that corresponds to this tree.

b) Show the result of performing insert(30).