Tutorial 02 - Priority Queues and More Asymptotic Analysis

CS 240E Winter 2022

University of Waterloo

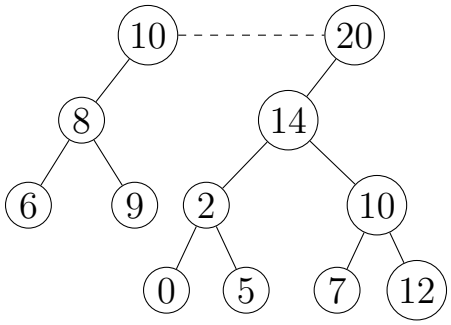
Monday, January 17, 2022

1. **Recursion:**
   Consider the following recursion: $T(0) = 0$,

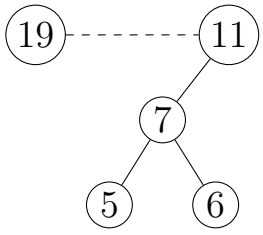
   $$T(n) = n + 1 + \min_{0 \le i \le n-1} \{T(i) + T(n - i - 1)\} \qquad \text{for } n \ge 1.$$

   Show that $T(n) \ge (n + 1)\log(n + 1)$. Hint: convince yourself that $f(x) = x \log x$ is convex.

2. **Binomial Heaps:**
   Perform the following operations on the binomial heap below, in order:

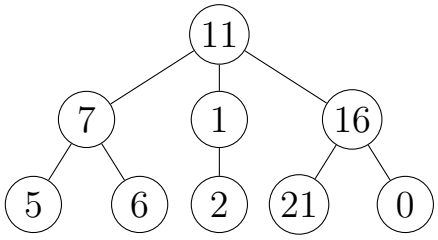
   - Insert a node with key 4.
   - Perform merge with the following binomial heap:

   - Call deleteMax.

3. **Multi-Way Tree:**
   Let T be a multi-way tree, i.e., nodes can have arbitrarily many children.

a) There is a simple way to convert a multi-way tree T into a binary tree T': each node of T also becomes a node in T', its leftmost child in T becomes the left child in T', and its sibling to the right in T becomes the right child in T'. Show the binary tree that you get in this way if you start with the following multi-way tree:



b) For which binary trees T' is there a multiway tree T that it corresponds to? Justify your answer by explaining how you would convert such a binary tree T' into a multiway tree T.

c) Assume T' is a flagged tree that satisfies the order-property of binomial heaps. What order-property does the corresponding multiway tree T have?