

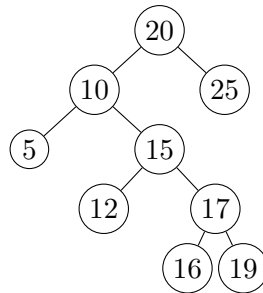
Tutorial 05 - Splay Trees & Re-orderings  
 CS 240E Winter 2022  
 University of Waterloo  
 Monday, February 7th, 2022

1. **Splay Trees:**

Given the following splay tree  $S$ , calculate its potential using the potential function

$$\Phi(i) := \sum_{v \in S} \log n_v^{(i)},$$

where  $n_v^{(i)}$  is the number of nodes in the subtree rooted at  $v$  after  $i$  operations, including  $v$  itself. Insert the key 18. Calculate the new potential. Verify that the difference between the potential difference is less than  $4 \log n - 2R + 2$ , where  $R$  is the number of rotations.



2. **Static Ordering:**

Let  $A$  be an unordered array with  $n$  distinct items  $k_0, \dots, k_{n-1}$ . Give an asymptotically tight  $\Theta$ -bound on the expected access cost if you put  $A$  in the optimal static order for the following probability distributions:

(a)  $p_i = \frac{1}{n}$  for  $0 \leq i \leq n - 1$

(b)  $p_i = \frac{1}{2^{i+1}}$ , for  $0 \leq i \leq n - 2$ ,  $p_{n-1} = 1 - \sum_{i=0}^{n-2} p_i = \frac{1}{2^{n-1}}$

3. **Dynamic Orderings:**

Consider a linked list with the keys  $k_1, k_2, \dots, k_n$  in that order. Give a sequence of  $n$  searches such that the Move-To-Front heuristic uses  $O(n)$  comparisons while the Transpose heuristic uses  $\Omega(n^2)$  comparisons.