Tutorial 08 - Range Search
Solutions
CS 240E Winter 2022
University of Waterloo
Monday, March 7th, 2022

1. **Burrows-Wheeler Transform:**

   a) For arbitrarily large N, construct a set of points such that the quad-tree has at least N nodes, and construct a range-search such that all nodes are visited, and not a single point gets returned.

   Solution Idea.

   Place points at $(0, y)$ for $y = 0, \cdots, 2^l - 1$, where $2^l > N$. This violates general position, so let the ith x-coordinate be $\frac{1}{2i+1}$. Then each y-coordinate is less than $\frac{1}{2}$, and each point is in general position. The bounding box will be $[0, 2^l)^2$. Also, the side length of each square containing the points is 1, since it takes l subdivisions to separate all the points. Then the query rectangle $[0.5, 2^l)^2$ intersects all regions containing the points, so all nodes are visited. But clearly no point is returned as no point has an x-coordinate of at least 0.5.

   b) Assume that T is a quad-tree (with at least two points) such that during some range-search, there is at least one outside node and at least one inside-node. The example from Module 8 (slide 11) satisfies this. What is the minimum possible height of T? The example has height 3, so the question is asking whether height 3 is always required, or whether this could also happen with height 2 or even height 1.

   Solution Idea.

   Recall that an inside node is a node whose region is completely contained in the query rectangle while an outside node is a node whose region is disjoint from the query rectangle.

   You cannot construct a quad-tree satisfying the constraints with height 0, since then there could only be one point, and it would either be an inside-node, an outside-node, or a boundary node, depending on how the query rectangle intersects the boundary region.

You can construct a quad-tree satisfying the constraints with height 1. For instance, take the points $(1, 1), (3, 3)$, which have the bounding box $[0, 4)^2$. Then take the query rectangle $[0, 2)^2$. It completely contains the region of the point $(1, 1)$ but is disjoint from that of $(3, 3)$ (which is $[2, 4)^2$).