University of Waterloo
An Old CS 466/666 Midterm Examination
Time: 50 minutes **Instructor: J. I. Munro**

1. [13 marks] Short answers

(a) [4 marks] We are given a set of n probabilities of accessing elements in a tree and compute the optimal search tree. As it turns out the probabilities are all the same. What is the expected cost in the optimal search tree? (Give the lead term including the constant of proportionality)

(b) [4 marks] Suppose you have coins in denominations of 1, 4 and 6 units. Does the greedy algorithm (repeatedly giving the largest coin possible without going over the required amount) always minimize the number of coins needed to produce a given amount of change? Justify this claim.

(c) [5 marks] In the linear median of Blum et al (as discussed in class), we sort elements into list of some odd constant length (the text used 5, we used 15). We then compute the median of medians and use it to reduce the number of candidates for the selection we are to perform. This leads to an O(n) method. Suppose we had lists of length 3 instead (of 5 or 15), what would be the runtime of the algorithm. Briefly justify your answer (no formal proof required)

2. [13 marks] Ranking in a Tableau **(NOT covered in 2008, but give it a try)**

Suppose you are given a Young tableau of size n by n containing n^2 elements. In such a structure all rows and columns are sorted in increasing order. All elements are distinct and the only operation you may use on elements is to compare them.

Problem: Given the tableau and another value x, you are to efficiently determine how many elements in the tableau are smaller than x.

(a) [5 marks] Sketch an algorithm (pseudocode is fine, but a less formal description is also acceptable if clear) to solve this problem. (Credit will depend on the efficiency, up to Θ notation, of the algorithm)

(b) [3 marks] What is the runtime, of your method. Use Θ notation.

(c) [5 marks] Give a lower bound on the number of comparisons needed for any possible algorithm to solve this problem. Justify this claim.

3. [13 marks]] Moo

A cow comes up to a fence, at a point we will call the origin. She remembers there is a hole somewhere in the fence, but does not recall where. She walks to the right (1 unit distance, a cowlometer if you like), then returns to the origin and continues 2 units to the left; then back to origin and 4 units right; back and 8 units left. She keeps doubling this maximum distance from the origin until she finds the hole in the fence. Your task is to show that she is indeed a smart cow and travels not all that much more than the minimum distance she has to walk.

(a) [5 marks] Give a recurrence equation that describes how far she has traveled *up to and including* her k^{th} attempt. (The first attempt is 1 unit right, the second is 1 back to the origin then 2 left; this gives a total of 4 units up to and including the second attempt)

(b) [5 marks] The cow is rather nearsighted, so the hole could be barely past (say ϵ past) the point at which she stops; but she would return to the origin, go twice the distance in the other direction and then return again and stop at the hole. Suppose this distance is $2^r + \epsilon$ from the origin. How far will she actually walk?

(d) [3 marks] Assuming she makes several attempts before finding the hole, what is maximum value of the ratio of distance walked to distance of the hole from the origin?

4. [11 marks] In the hull

Suppose you are given a convex polygon. The vertices are given in a sequence starting with the lowest y coordinate (and the least x coordinate if there is a tie) followed by the other nodes in the order they are encountered by following the polygon in a counter clockwise direction (increasing angle from the first point). Your task is to determine whether a given point, p, is in this polygon.

(a) [6 marks] Sketch an algorithm for this task. Again credit will depend on the efficiency of the algorithm, but any correct method will receive some credit.

(b) [5 marks] What is the runtime of your algorithm? Justify this claim (a phrase or short sentence will do).