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## Game Theory

Agents 1 and 2 play Split-the-Dollar. Each agent simultaneously name shares that they want, where $s_1$ is agent 1's requested share and $s_2$ is agent 2's requested share. If $s_1 + s_2 \leq 1$ then both agent gets their requested share. If $s_1 + s_2 > 1$ then both agents get zero. What are the pure strategy Nash equilibria in this game?

## Mechanism Design

(Thanks to K Leyton-Brown) Suppose you have some object that each of $n$ agents desires, but which you do not value. Assume that each agent $i$ values it at $v_i$ with $v_i$'s drawn independently and uniformly from some real line positive interval, say (0,$10^{100}$]. Although you do not desire the object and do not care about the actual values of the $v_i$'s, you need to compute $\sqrt{v_i}$ for each $i$.

Unfortunately, you face two problems. First, agents are not inclined to just reveal to you anything about their $v_i$'s. Second, your computer is costly to operate. It costs you 1 unit to determine the greater of two values, 2 units to perform any basic arithmetic operation $(+, -, \times, )$, and anything more complicated (say $\sqrt{x}$) costs 20. The (accurate) current time of day can be observed without cost.

1. How much does it cost to compute $\sqrt{v_i}$ for each $i$ using a straightforward VCG mechanism? When computing cost, ignore the revenue that the auction will generate.

2. Your answer above gives an upper bound on the cost of computing the square roots of the valuations. Design an incentive-compatible, dominant strategy ("strategy-proof") direct mechanism that will allow you to compute all $\sqrt{v_i}$ at minimal cost. Assume that the agents can do computations for free. Make sure that you specify all the components of the mechanism: players, actions, outcomes, mappings from actions to outcomes. Explain why your mechanism is strategy-proof. Specify the algorithms that you will use to implement those mappings. Give your mechanism's total computation cost (or an upper bound on it). You do not need to prove that your mechanism has minimal cost. (Hint: Think about the revelation principle.)

3. In the previous part you were restricted to direct mechanisms. Show that an *indirect* (multi-stage) mechanism can achieve even lower cost.

## Sponsored Search

In class we discussed the auction mechanism that is used for sponsored search. Is this auction (called the Generalized Second Price auction), an efficient and truthful auction? If your answer is

yes, provide a proof. If your answer is no, explain why not.