Assignment Guidelines.

- This assignment covers material in Module 2.
- Submission details:
  - Solutions to these questions must be placed in files a01q1.rkt, a01q2.rkt, a01q3.rkt, and a01q4.rkt, respectively, and must be completed using Racket Intermediate Student.
  - All solutions must be submitted to MarkUs. No solutions will be accepted through email, even if you are having issues with MarkUs.
  - Verify using MarkUs and your basic test results that your files were properly submitted and are readable on MarkUs.
  - For full style marks, your program must follow the CS115 Style Guide.
  - Be sure to review the Academic Integrity policy on the Assignments page.
  - For the design recipe, helper functions only require a purpose, a contract and an example.
- When a function returns an inexact answer, use a tolerance of 0.0001 in your tests.
- Restrictions:
  - Unless the question specifically describes exceptions, you are restricted to using the functions and special forms covered in or before Module 2.
  - Read each question carefully for additional restrictions.
- The solutions you submit must be entirely your own work. Do not look up either full or partial solutions on the Internet or in printed sources.
1. Design.

Write the design recipe, omitting the implementation, for a function \((\text{pi-digits } n)\).
It returns an approximation of \(\pi\), correct to \(n\) digits.

Catalan numbers, which have many applications in Combinatorics, are described here:
https://en.wikipedia.org/wiki/Catalan_number

Write the design recipe, omitting the implementation, for a function \((\text{catalan } n)\). It returns the \(n\)th Catalan number.

2. Areas.

Write a function \((\text{area-circle } r)\) that returns the area of a circle of radius \(r\).
For example,
\[\text{(area-circle 2)} \Rightarrow \#i12.566370614359172\]

Use the built in constant \(\pi \Rightarrow \#i3.141592653589793\) where needed.

Write a function \((\text{area-triangle } b \ h)\) that returns the area of a triangle with base \(b\) and height \(h\).
For example,
\[\text{(area-triangle 0.5 6)} \Rightarrow 1.5\]

The “usual” equation for the area of a triangle is useful only if the base and height are known. Sometimes
the three side lengths are known, but not the height. In this case, it is easier to use Heron’s Formula:
Given a triangle with side lengths \(a\), \(b\), and \(c\), the area of the triangle is
\[A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}\]
where \(s = \frac{a+b+c}{2}\) is the semi-perimeter of the triangle.

Write a function \((\text{area-heron } a \ b \ c)\) that returns the area of a triangle with side lengths \(a\), \(b\), and \(c\).
For example,
\[\text{(area-heron 4 13 15)} \Rightarrow 24\]
\[\text{(area-heron 2 2 2)} \Rightarrow (\text{sqrt } 3) \Rightarrow \#i1.7320508075688772\]

Note: for any real triangle, the sum of any two sides is greater than the remaining one. This is proved
in Euclid’s Elements, Book I, Proposition 20.
You cannot construct a triangle with side lengths 5, 2, and 2, for example.
Your function will give an imaginary result if you try:
\[\text{(area-heron 5 2 2)} \Rightarrow 0+3.75i\]
Imaginary numbers are not a part of this course, so work with real triangles. This would be a good
place to add a Requires.
3. Quadratic Equation.

The roots of a quadratic equation of the form \( y = ax^2 + bx + c \) are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

The portion \( b^2 - 4ac \) is sometimes called the **discriminant**.

Exercise

Write two functions: \((\text{root-plus } a \ b \ c)\) and \((\text{root-minus } a \ b \ c)\). Each function consumes three \text{Num}, and returns a \text{Num}. \text{root-plus} returns the more positive root of the equation, and \text{root-minus} returns the more negative root.

For example,

- \((\text{root-plus } 1 -8 15) \Rightarrow 5\)
- \((\text{root-minus } 1 -8 15) \Rightarrow 3\)
- \((\text{root-plus } 3 -6 1) \Rightarrow \#i1.81649658...\)
- \((\text{root-minus } 3 -6 1) \Rightarrow \#i0.183503419...\)

If the discriminant is negative, you will be taking the square root of a negative number, yielding an imaginary result. Again, we may prefer to work only with real numbers.

It would be a good idea to require the inputs to result in a non-negative discriminant. (Alternatively, if you are familiar with imaginary and complex numbers, you may write tests to deal with such cases.)

4. Str.

Read the documentation in DrRacket on the functions \text{min} and \text{max}, and review the documentation on \text{Str}. Some of these functions will be required to complete this question.

Exercise

Write a function \((\text{pad3 } n)\) that consumes a \text{Nat} and returns a \text{Str}. The \text{Str} contains the digits of \( n \), with zeros added at the front to make it of length 3. Only the first three digits of numbers 1000 or greater are retained.

For example,

- \((\text{pad3 } 7) \Rightarrow "007"\)
- \((\text{pad3 } 42) \Rightarrow "042"\)
- \((\text{pad3 } 245) \Rightarrow "245"\)
- \((\text{pad3 } 3141592) \Rightarrow "314"\)

We will not discuss \text{cond} until Module 4. Do not use it on this assignment. The necessary effects can be achieved using some combination of \text{min}, \text{max}, and \text{Str} functions.