In Racket, select Language → Choose language → Intermediate student with lambda.

If you have not already, make sure you

- Read the Wikipedia entry on Higher-order functions.
So far we have written only functions that consume one or a few values, and may combine them in various ways.

More often we have a collection of data to process.

Racket is a dialect of **LISP**, which was originally designed for **LIS**t Processing.

Our principal way of grouping values is the list.
The word list comes from Old English “líste”, meaning a strip (such a strip of cloth or paper).

“Heis targe wip gold list He carf atvo.”
(Guy of Warwick, ca. 1330)

→ A strip of paper with items written on it.
→ An ordered collection of items.

We can make a list really easily:

(define wishes
  (list "comics" "turtle figures"
       "Donkey Kong" "Play-Doh Burger King"))

(define primes (list 2 3 5 7 11 13 17 19))
A value may be a list

Lists behave just like any other value.

We can define constants which are lists:

```
(define wishes
  (list "comics" "turtle figures"
        "Donkey Kong" "Play-Doh Burger King"))
```

```
(define primes (list 2 3 5 7 11 13 17 19))
```

We can have functions consume lists:

```
(length wishes) => 4
(first wishes) => "comics"
(rest wishes) => (list "turtle figures" "Donkey Kong" "Play-Doh Burger King")
```

We can have functions return lists:

```
(range 4 16 2) => (list 4 6 8 10 12 14)
(append (list 6 7 42) (list 3 5 15)) => (list 6 7 42 3 5 15)
```
Lists and the design recipe

In the design recipe, we specify the type of values in a list as follows:

- Use `(listof Type)` for a single type.
  - `(listof Nat)` describes a list containing zero or more `Nat`. E.g. `(list 6 7 42)`
  - `(listof Str)` describes a list containing zero or more `Str`. E.g. `(list "hello" "world")`

- If a list may contain more than one type, use `(listof (anyof Type1 Type2 ...))`.
  - `(listof (anyof Num Str))` describes a list containing zero or more values, each of which is either a `Num` or a `Str`. E.g. `(list 3.14 "pie" "forty-two" -17)`

- If a list is of known length and types, use `(list Type1 Type2 ...)`.
  - `(list Nat Str)` describes a list containing two values. The first value is a `Nat`, and the second value is a `Str`. E.g. `(list 6 "foo")`
  - `(list "foo" 6)` is not a `(list Nat Str)`. It is a `(list Str Nat)`. 
Transforming items in a list using map

We can *store* data in a list, but what can we *do* with them?

Use **map** to transform each item in a list, using a function.

\[
\text{map } F \left( \text{list } x_0 \ x_1 \ x_2 \ldots \ x_n \right) \Rightarrow \left( \text{list } (F \ x_0) \ (F \ x_1) \ (F \ x_2) \ldots \ (F \ x_n) \right)
\]

\[
\text{map } \text{sqr} \ \left( \text{list } 2 \ 3 \ 5 \right) \Rightarrow \left( \text{list } \text{sqr} \ 2 \ \text{sqr} \ 3 \ \text{sqr} \ 5 \right) \Rightarrow \left( \text{list } 4 \ 9 \ 25 \right)
\]

\[
\text{define} \ (\text{double } x) \ (+ \ x \ x)
\]

\[
\text{define} \ (\text{double-each } L)
\]

\[
\quad \text{map } \text{double} \ L
\]

\[
\text{double-each } \left( \text{list } 0 \ 1 \ 2 \ 3 \ 4 \right) \Rightarrow \left( \text{list } 0 \ 2 \ 4 \ 6 \ 8 \right)
\]
Strategy for working with \texttt{map}

To use \texttt{map} on a list of values of some type:
write a function that consumes \textit{one single value} of that type and transforms it as required.

I wish to transform each item in a list by $f(x) = 10\sqrt{x}$:

\begin{verbatim}
;; (10rootx n) return 10*sqrt(x)
;; 10rootx: Num -> Num
;; Examples:
(check-expect (10rootx 49) 70)

(define (10rootx x) (* 10 (sqrt x)))
\end{verbatim}

\begin{verbatim}
;; (10rootx-each L) return a list containing 10*sqrt(x) for each x in L.
;; 10rootx-each: (listof Num) -> (listof Num)
;; Requires: each value is \texttt{>= 0}
;; Examples:
(check-expect (10rootx-each (list 49 81 100)) (list 70 90 100))

(define (10rootx-each L) (map 10rootx L))
\end{verbatim}
To use `map` on a list of values of some type: write a function that consumes *one single value* of that type and transforms it as required.

Exercise

Digital signals are often recorded as values between 0 and 255, but we often prefer to work with numbers between 0 and 1.

Write a function `(squash-range L)` that consumes a `(listof Nat)`, and returns a `(listof Num)` so numbers on the interval `[0, 255]` are scaled to the interval `[0, 1]`.

`(squash-range (list 0 204 255)) => (list 0 0.8 1)`
Using range to build lists

(range start end step) returns the list that starts at start, and steps by step until just before it reaches end. This allows us to build new lists.

(range 4 10 1) => (list 4 5 6 7 8 9)
(range 4 10 2) => (list 4 6 8)
(range 20 8 -3) => (list 20 17 14 11)
(range 20 8 3) => '() ;; the empty list

To work with range and map:
1. get proper values from range; test it.
2. use map to transform these values as needed.

Exercise

Complete the function list-cubes.

;; (list-cubes n) return the list of cubes from 1*1*1 to n*n*n.
;; list-cubes: Nat -> (listof Nat)
;; Examples:
(check-expect (list-cubes 4) (list 1 8 27 64))
Summarizing a list using foldr

**range** lets us create a list, and **map** lets us transform each item. What if I want to my result to be a combination of the items in the list, instead of the entire list?

What is the total of all the values in $(\text{list} \ 6 \ 5 \ 8 \ 5 \ 14 \ 4)$?

$$(+ \ 6 \ (+ \ 5 \ (+ \ 8 \ (+ \ 5 \ (+ \ 14 \ 4))))))) \Rightarrow 42$$

To do this automatically, there is another function, **foldr**, meaning “fold right”.

$$((\text{foldr} \ F \ \text{base} \ (\text{list} \ \text{x0} \ \text{x1} \ ... \ \text{xn})) \Rightarrow (F \ \text{x0} \ (F \ \text{x1} \ (F \ ... \ (F \ \text{xn} \ \text{base})))) \))$$

What does this mean?

We combine items, starting from the right, each time creating a new item to combine with.

$$(\text{foldr} \ + \ 0 \ (\text{list} \ 6 \ 5 \ 8 \ 5 \ 14 \ 4))$$

$$\Rightarrow (+ \ 6 \ (+ \ 5 \ (+ \ 8 \ (+ \ 5 \ (+ \ 14 \ (+ \ 4 \ 0)))))))$$

$$\Rightarrow 42$$
Strategy for working with foldr

\[
\text{foldr } F \text{ base } (\text{list } x_0 x_1 \ldots x_n) \Rightarrow (F x_0 (F x_1 (F \ldots (F x_n \text{ base}))))
\]

1. Figure out what the answer is when the list is empty. Use this as the base.
2. Write a function that consumes two values, new and old, where new is a value from the list, and old is an answer.

For example: consider finding the sum of items in a (listof Num).

1. The sum of nothing is zero, so the base is 0.
2. To calculate the sum of a value and another sum, just add the two values.

\[
\begin{align*}
\text{(define (add a b) (+ a b))} \\
\text{(define (sum L) (foldr add 0 L))} \\
\text{(sum '()) => 0)} \\
\text{(sum (list 5 8 4)) => (add 5 (add 8 (add 4 0)))) => 17}
\end{align*}
\]

(We could use the built-in function +.)
Working with foldr

(\texttt{foldr} \ F \ \texttt{base} \ (\texttt{list} \ x_0 \ x_1 \ \ldots \ x_n)) \Rightarrow (F \ x_0 \ (F \ x_1 \ (F \ \ldots \ (F \ x_n \ \texttt{base}))))

1. Figure out what the answer is when the list is empty. Use this as the base.
2. Write a function that consumes two values, \texttt{new} and \texttt{old}, where \texttt{new} is a value from the list, and \texttt{old} is an answer.

Exercise

Write a function \texttt{prod} that returns the product of a (\texttt{listof Num}).

\texttt{(prod (list 2 2 3 5))} \Rightarrow \texttt{60}

Write a function that returns the number of odd numbers in a \texttt{listof Nat}.

Hint: read the documentation on \texttt{remainder}.

Can you do this using \texttt{map} and \texttt{foldr}? Just using \texttt{foldr}?
Exercises

Exercise

Experiment with \texttt{fold-sub}. Describe how it behaves, and why.

\begin{verbatim}
(define (fold-sub L) (foldr - 0 L))
(fold-sub (list 6 5 2)) => ?
\end{verbatim}

Exercise

Write a function \texttt{flatten} that consumes a \texttt{(listof (listof Any))} and returns a list containing all the values in the lists.

\begin{verbatim}
(flatten (list (list 1 2) (list 3 4) (list 7))) => (list 1 2 3 4 7)
\end{verbatim}

Hint: read the documentation on \texttt{append}.

Exercise

Read the documentation on \texttt{string-length}.

Write a function that returns the total length of all the values in a \texttt{(listof Str)}.
Exercises

Write a function that returns the average (mean) of a (listof Num).

(average (list 2 4 9)) => 5
(average (list 4 5 6 6)) => 5.25

Recall that (length L) returns the number of values in L.

The factorial function, \( n! \), returns the product of the numbers from 1 to \( n \). For example, \( 4! = 1 \times 2 \times 3 \times 4 = 24 \).

Write a function (factorial n) that returns \( n! \).

(factorial 5) => 120
(factorial 1) => 1
Write a function \( \text{(sum-square-difference } n) \) that consumes a \textit{Nat} and returns the difference between the square of the sum of numbers from 0 to \( n \), and the sum of the squares of those numbers.

\[
\text{(sum-square-difference 3) } \Rightarrow (- (\text{sqr } (+ 0 1 2 3)) (+ 0 1 4 9)) \Rightarrow 22
\]
Some simple things are annoying

If I wanted to, for example, double each item in a list:

```scheme
;; (double n) return 2*n.
;; double: Num -> Num
;; Examples:
(check-expect (double 3) 6)
(check-expect (double 0) 0)
```

```scheme
(define (double n) (* n 2))
```

```scheme
;; (double-each L) return L, with each value doubled.
;; double-each: (listof Num) -> (listof Num)
;; Examples:
(check-expect (double-each '()) '())
(check-expect (double-each (list 2 3 5)) (list 4 6 10))
```

```scheme
(define (double-each L)
  (map double L))
```

Half the work is the design recipe for a really simple function!
Tiny Functions with lambda

For short functions which are used just once, lambda lets us write anonymous functions.

;; (double-each2 L) return L, with each value doubled.
;; double-each2: (listof Num) -> (listof Num)
;; Examples:
(check-expect (double-each2 '()) '())
(check-expect (double-each2 (list 2 3 5)) (list 4 6 10))

(define (double-each2 L)
  (map (lambda (n) (* n 2)) L))

lambda is a function that returns a function.

(lambda (x) (+ x 7)) is a function with one parameter.

(map (lambda (x) (+ x 7)) (list 2 3 5)) => (list 9 10 12)

Exercise

Using lambda and map, but no helper functions, write a function that consumes a (listof Num) and returns a list containing the cube of each Num. (x^3)
Handling extra parameters with lambda

Suppose I wanted to add 5 to every item in a list:

;; (add-5 n) add 5 to n.
;; add-5: Num -> Num
(define (add-5 n) (+ n 5))

;; (add-5-each L) add 5 to each item in L.
;; add-5-each: (listof Num) -> (listof Num)
(define (add-5-each L) (map add-5 L))

(check-expect (add-5-each (list 3.2 6 8)) (list 8.2 11 13))

This works!

But now suppose I want to be able to add a different value to each. There’s a problem: if I add a parameter n to add-5-each, there is no way for that value to be available to add-5.
Handling extra parameters with lambda

We can fix it using lambda!

;;;; (add-n-each L n) add n to each item in L.
;;;; add-n-each: (listof Num) Num -> (listof Num)
(define (add-n-each L n)
 (map (lambda (x) (+ x n))
    L))

(check-expect (add-n-each (list 3.2 6 8) 6) (list 9.2 12 14))

This lambda function can use the value of n.

Exercise
Write a function (multiply-each L n). It consumes a (listof Num) and a Num, and returns the list containing all the values in L, each multiplied by n.

(multiply-each (list 2 3 5) 4) => (list 8 12 20)

Exercise
Write a function (add-total L) that consumes a (listof Num), and adds the total of the values in L to each value in L.

(add-total (list 2 3 5 10)) => (list 22 23 25 30)
A few details about lambda

Using `lambda` we can create a constant which stores a function.

```scheme
(define double (lambda (x) (* 2 x)))
```

```scheme
(double 5) => 10
```

(If you do this, you are creating a named function, so you must use the design recipe!)

You can use a `lambda` expression anywhere you need a function:

```scheme
(((lambda (x y) (+ x y y)) 2 5) => 12
```

Anything that can go in a function can go in a `lambda`, even another `lambda`:

```scheme
(((lambda (x y)
    ((lambda (z) (+ x z)) y)) 4 5)
```
Multi-argument map

So far we have used map only with functions that consume a single value: like (map F L), where F is a single-parameter function and L is a list. But map can do so much more!

map works with any number of lists, all of the same length: (map F L1 L2…)

For example, we can implement vector addition very easily:

;; A Vector is a (listof Num)

;; (vector-add a b) return the vector sum of a and b.
;; vector-add: Vector Vector -> Vector
;; Requires: a and b are of equal length.
;; Examples:
(check-expect (vector-add (list 2 3 3) (list 7 4 1)) (list 9 7 4))

Exercise

Write a function (double-add a b) that consumes two Vector and returns twice the vector sum of them.

(double-add (list 2 3 3) (list 7 4 1)) => (list 18 14 8)

! Use map only once!
Recall that the distance of a point \((x, y)\) from \((0, 0)\), from the Pythagorean theorem, is \(\sqrt{x^2 + y^2}\).

You may use the \texttt{sqrt} function to compute the square root. \((\texttt{sqrt 4}) \Rightarrow 2\).

Write a function \((\text{distances} \; \text{xs} \; \text{ys})\) that consumes two lists: the first contains \(x\) values, and the second contains \(y\) values. The output is a list containing the distance of each point \((x, y)\) from \((0, 0)\).

\((\text{distances} \; \text{list} \; 3 \; 0 \; 2) \; \text{list} \; 4 \; 7 \; 2)) \Rightarrow \text{list} \; 5 \; 7 \; \#i2.828427)
Here is one solution:

;; (distance x y) return the distance from (x, y) to the origin.
;; distance: Num Num -> Num
;; Example:
(check-expect (distance 3 4) 5)

(define (distance x y)
  (sqrt (+ (sqr x) (sqr y))))

;; (distances xs ys) return the list of distances for each of xs, ys.
;; distances: (listof Num) (listof Num) -> (listof Num)
;; Example:
(check-within (distances (list 3 0 2) (list 4 7 2))
  (list 5 7 2.8284) 0.0001)

(define (distances xs ys)
  (map distance xs ys))
Multi-argument map

Suppose we have two \texttt{(listof Str)}: one of first names, and one of matching last names:

\begin{verbatim}
(define gnames (list "David" "James" "Douglas" "Burt" "Joseph"))
(define snames (list "Johnston" "Downey" "Wright" "Matthews" "Hagey"))
\end{verbatim}

Exercise

Complete join-names.

\begin{verbatim}
;; (join-names G S) Make a list of full names from G and S.
;; join-names: (listof Str) (listof Str) -> (listof Str)
;; Example:
(check-expect (join-names gnames snames)
  (list "David Johnston" "James Downey" "Douglas Wright"
    "Burt Matthews" "Joseph Hagey"))
\end{verbatim}
Module Summary

- Start storing information in lists, and describe lists in contracts.
- Transform list values using `map`, and `foldr`.
- Construct new lists using `range`, especially in combination with `map`.
- Use `foldr` to combine a list to a single value. This can be especially powerful when combined with `map`.
- Be able to use `lambda` functions in combination with `map` and `foldr`.
- Understand the use of `anyof` and be able to use it in your design recipes.

Before we begin the next module, please

- Read *How to Design Programs*, Section 4