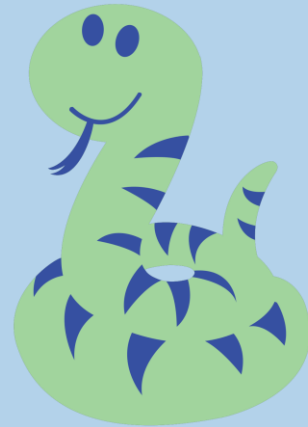


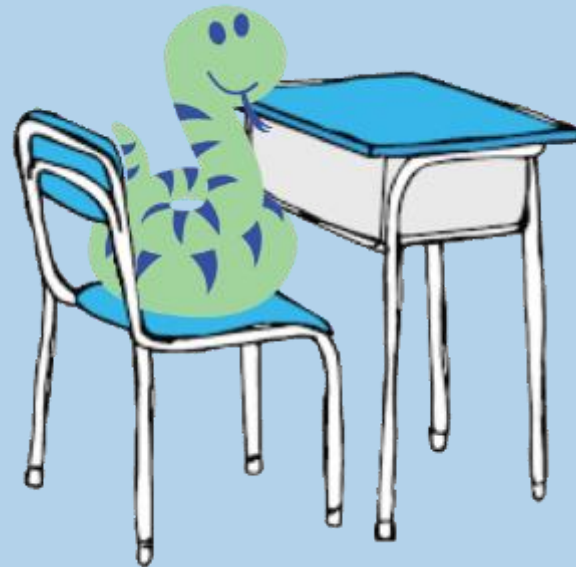
TUTORIAL 7

EFFICIENCY



REMINDER

- Assignment 6 is due next Wednesday, 10 AM.
- Final is on April 15th, at 4:00 pm.



CLICKER QUESTION 1

How was the midterm?

- A. Pretty smooth and there was enough time
- B. Overall easy but long
- C. A bit difficult in some questions but overall reasonable length and difficulty
- D. Overall difficult.
- E. 😞

RUNTIME REVIEW

- Look at the “worst case” scenario (*i.e. longest runtime*)
- Assume function works (*i.e. will not return an error when you run it*)
- Based on the assumptions learned in class (and in the modules)

RUNTIME REVIEW

- $O(1)$ – Constant
 - does not depend on the size of the input
 - Comparison operations: `>`, `>=`, `<`, `<=`, `==`
 - Value assignment: (i.e. `x = 4`)
 - For numbers:
 - Numeric operations: `+`, `*`, `/`, `-`, `%`, `//`
 - `max`, `min`
 - For list `L`:
 - `L[0]`, `L[1]`, `L[2]`, ..., `len(L)`...
 - `L.append(4)`...
- $O(n)$ – Linear
 - depends on the size of the input
 - For list `L` (assume the length of `L` is `n`):
 - `L[1:]`, `max(L)`, `L + L`, `sum(L)`, `L.remove(0)`...
 - `list(map(lambda x: x+1, L))`

RUNTIME REVIEW

- $O(n^2)$ – Quadratic

- time proportional to square of size of the input
- Be careful of abstract functions:

- `list(map(lambda k: list(range(k)), list(range(n))))`

- $O(2^n)$ – Exponential

- As size of input increases by 1, the run time doubles
- example: Module 5, Slide 15: `fib`

USEFUL SUMMATIONS

- $\sum_{i=1}^n 1 = O(n)$
- $\sum_{i=1}^n i = O(n^2)$
- $\sum_{i=1}^n n = O(n^2)$
- $\sum_{i=1}^n \sum_{j=1}^n 1 = O(n^2)$

RUNTIME EXAMPLE 1

```
# Let n = len(L)
def fn(L):
    ans = []
    for x in L:
        if x[0]=='A':
            ans.append(x)
    return ans
```

Count steps for:

- Assign [] to ans
- Loop:
 - Number of Iterations
 - Asymptotic run time of the body of loop:
 - Check if `x[0] == 'A'`
 - `ans.append(x)`
- Return ans
- $\sum_{i=1}^n 1 = O(n)$

RUNTIME EXAMPLE 2

Let n be a natural number

```
def fn(n):  
    ans = 1  
    collection = list(range(2*n))  
    for x in collection:  
        if x%10==1:  
            ans = ans + 1  
        else:  
            ans = ans + 2  
    return ans
```

Count steps for:

- Assign 1 to ans
- `list(range(2*n))`
- Assign value to collection
- Loop:
 - Number of Iterations
 - Asymptotic run time of the body of loop:
 - Calculate `x%10`
 - Check if `x%10 == 1`
 - `ans = ans + 1`
(or `ans = ans + 2`)
- Return ans
- $O(n) + \sum_{i=1}^n 1 \Rightarrow O(n) + O(n) \Rightarrow O(n)$

RUNTIME EXAMPLE 3

```
def fn(n):  
    if n % 2 == 0:  
        return "outcome1"  
    elif n % 3 == 0:  
        return "outcome2"  
    elif n % 5 == 0:  
        return "outcome3"  
    else:  
        return "outcome4"
```

Count steps for:

- Calculate $n\%2$
- Compare it with 0
- Calculate $n\%3$
- Compare it with 0
- Calculate $n\%5$
- Compare it with 0
- Return the answer
- $O(1)$

RUNTIME EXAMPLE 4

```
def fn(L):  
    return len(list(filter(lambda x:  
                           x == max(L), L)))
```

Count the steps for:

- `max()`
- **Check** `x == max(L)`
- **Filter**
- `len()`
- $O(n) * O(n) \Rightarrow O(n^2)$

CLICKER QUESTION 1A

a) Determine the worst-case run-time in terms of n , where $n = \text{len}(\text{loi})$

```
def evens(loi):  
    return list(filter(lambda x: x%2 == 0, loi))
```

- A. $O(1)$
- B. $O(n)$
- C. $O(n^2)$
- D. $O(2^n)$

CLICKER QUESTION 1B

b) Determine the worst-case run-time in terms of n

```
def create_number_lists(n):  
    total = []  
    while n != 0:  
        i = 0  
        sublist = []  
        while i < n:  
            sublist.append(i)  
            i = i + 1  
        total.append(sublist)  
        n = n - 1  
    return total
```

- A. $O(1)$
- B. $O(n)$
- C. $O(n^2)$
- D. $O(2^n)$

ITEM DEFINITION

A `Card` is a list of length 2 where

- the first item is an integer between 1 and 13, inclusive, representing the `value` of the card, and
- the second item is a string ("hearts", "spades", "clubs", or "diamonds") representing the `suit` of the card.

Example: `[1, "hearts"]` represents the ace of hearts



LOOP: TUTORIAL 4 Q1

Use loops, write a function `create_cards` that consumes two lists with same length, which are a list of card values (integers between 1 and 13), and a list of suit values (one of the four suit strings), and returns a list of `Card`, created pair-wise from the consumed lists (`values and suits`).

For example,

```
create_cards([4,1,10],["hearts", "diamonds", "clubs"])  
=>[[4,"hearts"], [1, "diamonds"], [10, "clubs"]]
```

WHAT IS THE RUNTIME OF CREATE_CARDS?

```
def create_cards(values, suits):  
    acc = []  
    for i in range(len(values)):  
        acc.append([values[i], suits[i]])  
    return acc
```

- A. $O(n)$
- B. $O(n^2)$
- C. $O(2^n)$

WHAT IS THE RUNTIME IF WE USE ABSTRACT LIST FUNCTION?

```
def create_cards(values, suits):  
    return list(map(lambda x, y: [x, y], values, suits))
```

LOOP: TUTORIAL 5 Q2

Using loops, write a function `count_max` that consumes a **nonempty list** of integers `alon` and returns the number of times the largest integer in `alon` appears.

Note: - `max` and `L.count()` cannot be used in this question.

- Your function can only pass through the list once

For example,

```
count_max([1, 3, 5, 4, 2, 3, 3, 3, 5]) => 2
```

since the largest element of the list, 5, appears twice. Your function should pass through the list only once.

WHILE LOOP SOLUTION

```
def count_max(alon):
    current_max = alon[0]
    max_occur = 0
    while L != []:
        if L[0] > current_max:
            current_max = alon[0]
            max_occur = 1
        elif L[0] == current_max:
            max_occur += 1
        L = L[1:]
    return max_occur
```

What is the runtime?

Let us solve the question with
runtime $O(n)$!

FOR LOOP SOLUTION

```
def count_max(lon):  
    current_max = lon[0]  
    max_occur = 0  
    for each in lon:  
        if each > current_max:  
            current_max = each  
            max_occur = 1  
        elif each == current_max:  
            max_occur += 1  
    return max_occur
```