TUTORIAL 8

EFFICIENCY
REMINDERS

• Assignment 7 due Wednesday, March 20th, at 10:00 AM
RUNTIME REVIEW

- Look at the “worst case” scenario (i.e. longest runtime)
- Assume function works (i.e. will not return an error when you run it)
- Based on the assumptions learned in class (and in the modules)
RUNTIME REVIEW

• O(1) – Constant
  – does not depend on the size of the input
  – For numbers:
    • Numeric operations: +, *, /, -, %, //
    • max, min
  – For list L:
    • L[0], len(L)...
    • L.append(4)...

• O(n) – Linear
  – depends on the size of the input
  – For list L(assume the length of L is n):
    • L[1:], max(L), L + L, sum(L), L.remove(0)...
    • list(map(lambda x: x+1, L))
RUNTIME REVIEW

• $O(n^2)$ – Quadratic
  – time proportional to square of size of the input
  – Be careful of abstract functions:
    • `list(map(lambda k: list(range(k)), list(range(n))))`

• $O(2^n)$ – Exponential
  – As size of input increases by 1, the run time doubles
  – example: Module 5, Slide 15: `fib`
RECURRANCE RELATIONS

• $T(n) = O(1) + T(n - 1) \rightarrow O(n)$

• $T(n) = O(n) + T(n - 1) \rightarrow O(n^2)$

• $T(n) = O(1) + T(n/2) \rightarrow O(\log n)$

• $T(n) = O(n) + 2T(n/2) \rightarrow O(n \log n)$

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• $T(n) = O(1) + T(n - 1) + T(n - 2) \rightarrow O(2^n)$

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USEFUL SUMMATIONS

- $\sum_{i=1}^{n} 1 = O(n)$
- $\sum_{i=1}^{n} i = O(n^2)$
- $\sum_{i=1}^{n} n = O(n^2)$
- $\sum_{i=1}^{n} \sum_{j=1}^{n} 1 = O(n^2)$
# Let n = len(L)
def fn(L):
    if L == []:
        return 0
    else:
        return 1 + fn(L[1:])

Count steps for:
• Compare L with []
• Calculate L[1:]
• Call fn recursively on a list of length n-1
• Add 1 to the recursive call of fn

• $T(n) = O(n) + T(n-1) \Rightarrow O(n^2)$
# Let $n = \text{len}(L)$

def fn(L):
    ans = []
    for x in L:
        if x[0]=='A':
            ans.append(x)
    return ans

Count steps for:
- Assign $\text{[]}$ to $\text{ans}$
- Loop:
  - Number of Iterations
  - Asymptotic run time of the body of loop:
    - Check if $x[0] == \text{'}A\text{'}$
    - $\text{ans.append(x)}$
- Return $\text{ans}$
- $\sum_{i=1}^{n} 1 = O(n)$
# Let n = len(L)
def fn(L):
    L1 = L[0::2]
    if L == []:
        return []
    else:
        return fn(L1)

Count steps for:

- L1 = L[0::2]
- Compare L with []
- Call fn recursively on a list of length n // 2
- T(n) = O(n) + T(n/2) => O(n)
def fn(n):
    if n % 2 == 0:
        return "outcome1"
    elif n % 3 == 0:
        return "outcome2"
    elif n % 5 == 0:
        return "outcome3"
    else:
        return "outcome4"

Count steps for:
- Calculate \( n \% 2 \)
- Compare it with 0
- Calculate \( n \% 3 \)
- Compare it with 0
- Calculate \( n \% 5 \)
- Compare it with 0
- Return the answer
- \( O(1) \)
QUESTION 2 - QUICKSORT

Consider a different way of sorting a list L of distinct integers:
- Let \( x \) be the first element of the list
- Let \( \text{lst1} \) be all the elements in the list smaller than \( n \)
- Let \( \text{lst2} \) be all the elements in the list larger than \( n \)
- Recursively quicksort \( \text{lst1} \) and \( \text{lst2} \)
  - \( \text{lst1} + [x] + \text{lst2} \)

Write a function \texttt{quicksort} which consumes a list of distinct integers, \( \text{lst} \), and sorts it using the quicksort algorithm.


**EXAMPLE**

quicksort([2,3,1,4,0])

→ quicksort([1,0]) + [2] + \\
    quicksort([3,4])

→ ( quicksort([0]) + [1] ) + [2] + \\
    ( [3] + quicksort([4]) )

→ ([0] + [1]) + [2] + ([3] + [4])

→ [0, 1] + [2] + [3, 4]

→ [0, 1, 2, 3, 4]
RUNTIME OF QUICKSORT

- **Worst case runtime**:  
  - $T(n) = O(n) + T(n-1) \Rightarrow O(n^2)$  
  - The list is already sorted

- In practice, quicksort can avoid the worst case most of the time, and, on average, runs in $O(n \log n)$ time.