

# Functions

## Readings:

- HTDP, sections 1-3
- Thrival and Style guides

## Topics:

- Programming language design
- The DrRacket environment
- Values, expressions, & functions
- Defining functions
- Programming in DrRacket

PL Design ○○○○ 1/33	DrRacket ○○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--	--------------------------------

# Programming language design

**Imperative:** based on frequent changes to data

- Examples: machine language, Java, C++, Turing, VB

**Functional:** based on the computation of new values rather than the transformation of old ones.

- Examples: Excel formulas, LISP, ML, Haskell, Erlang, F#, Mathematica, XSLT, Clojure.
- More closely connected to mathematics
- Easier to design and reason about programs

PL Design ●○○○ 2/33	DrRacket ○○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--	--------------------------------

# > Racket

## Attributes:

- a functional programming language
- minimal but powerful syntax
- small toolbox with ability to construct additional required tools
- interactive evaluator
- graduated set of teaching languages are a subset of Racket

## Background:

- used in education and research since 1975
- a dialect of Scheme; descendant of Lisp

PL Design ●○○○ 3/33	DrRacket ○○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--	--------------------------------

## > Functional vs. imperative

Functional and imperative programming share many concepts.

However, they require you to think differently about your programs.

If you have had experience with imperative programming, you may find it difficult to adjust initially.

By the end of CS 136, you will be able to express computations in both these styles, and understand their advantages and disadvantages.

PL Design ○○●○ 4/33	DrRacket ○○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--------------------------------------	--------------------------------

## Values, expressions, & functions (intro)

**Values** are numbers or other mathematical objects.

Examples: 5, 4/9,  $\pi$ .

**Expressions** combine values with operators and functions.

Examples:  $5 + 2$ ,  $\sin(2\pi)$ ,  $\frac{\sqrt{2}}{100\pi}$ .

**Functions** generalize similar expressions.

Example:

$$3^2 + 4(3) + 2$$

$$6^2 + 4(6) + 2$$

$$7^2 + 4(7) + 2$$

are generalized by the function

$$f(x) = x^2 + 4x + 2.$$

PL Design ○○●○ 5/33	DrRacket ○○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--------------------------------------	--------------------------------

## The DrRacket environment

- Designed for education, powerful enough for “real” use
- Sequence of language levels keyed to textbook
- Includes good development tools
- Two windows: Interactions (now) and Definitions (later)
- Interactions window: a read-evaluate-print loop (REPL)

PL Design ○○○○ 6/33	DrRacket ●○○○	Values, expressions, & functions ○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--------------------------------------	--------------------------------

## Exercise 1

Install DrRacket [www.racket-lang.org/download/](http://www.racket-lang.org/download/) on your computer.

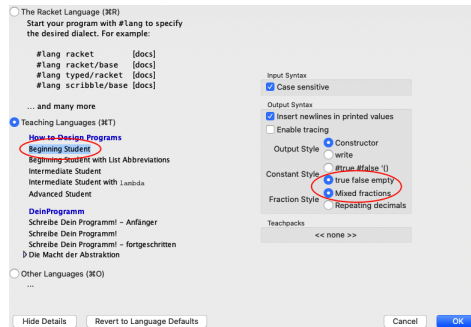
Choose Distribution: **Racket** and Variant: **Regular**.

If you have any trouble, ask in the discussion forum.

### > Setting the language in DrRacket

CS 135 will progress through the Teaching Languages starting with *Beginning Student*. **Follow steps 3 - 5 each time you change the language.**

- 1 Under the *Language* tab, select *Choose Language ...*
- 2 Select *Beginning Student* under *Teaching Languages*
- 3 Click the *Show Details* button in the bottom left
- 4 Under *Constant Style*, select *true false empty*
- 5 Under *Fraction Style*, select *Mixed fractions*



PL Design ○○○○ 7/33	DrRacket ○○●○	Values, expressions, & functions ○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--	--------------------------------

### > Values (numbers) in Racket

- Integers in Racket are unbounded.
- Rational numbers are represented exactly:  $2, 3\frac{1}{7}$
- Expressions whose values are not rational numbers are flagged as being **inexact**:  
 $(\text{sqrt } 2) \Rightarrow \#i1.414213562370951$ .  
We will not use inexact numbers much.

```
Welcome to DrRacket, version 7.3 [3m].  
Language: Beginning Student [custom];  
memory limit: 128 MB.  
> (expt 2 500)  
32733906078961418700131896968275991 2  
52216642046043064789483291368096133 2  
7964046745548832709232590415715088 2  
66841275600710092172565458853930533 2  
28527589376  
> (/ 3 5.55)  
20  
37  
> (sqrt 5)  
#i2.23606797749979  
>
```

We will, in time, add other values: symbols, Booleans, strings, etc.

PL Design ○○○○ 8/33	DrRacket ○○●○	Values, expressions, & functions ○○○○○○○○○○○○○○○○ 02: Functions	Defining functions ○○○○○○○○○○○○○○○○	Programming ○○○○○ CS 135
---------------------------	------------------	---	--	--------------------------------

## > Functions in mathematics

Definitions:  $f(x) = x^2$ ,  $g(x, y) = x + y$

These definitions consist of:

- the name of the function (e.g.  $g$ )
- its **parameters** (e.g.  $x, y$ )
- an algebraic expression using the parameters as placeholders for values to be supplied in the future

## > Function application

Definitions:  $f(x) = x^2$ ,  $g(x, y) = x + y$

An **application** of a function supplies **arguments** for the **parameters**, which are substituted into the algebraic expression.

An example:  $g(1, 3) = 1 + 3 = 4$

The arguments supplied may themselves be applications.

Example:  $g(g(1, 3), f(3))$

## > Function application

Definitions:  $f(x) = x^2$ ,  $g(x, y) = x + y$

We **evaluate** each of the arguments to yield values.

Evaluation by **substitution**:

$$g(g(1, 3), f(3)) =$$

$$g(1 + 3, f(3)) =$$

$$g(4, f(3)) =$$

$$g(4, 3^2) =$$

$$g(4, 9) = 4 + 9 = 13$$

## > Many possible substitutions

Definitions:  $f(x) = x^2$ ,  $g(x, y) = x + y$

There are many mathematically valid substitutions:

$$g(g(1, 3), f(3)) = g(1 + 3, f(3))...$$

$$g(g(1, 3), f(3)) = g(g(1, 3), 3^2)...$$

$$g(g(1, 3), f(3)) = g(1, 3) + f(3)...$$

**Canonical form:** a natural unique representation of an object, or a preferred notation for some object

We'd like a **canonical form** for two reasons:

- Easier for us to think about
- When we extend this idea to programming, we'll find cases where different orderings result in different values

PL Design  
○○○○  
12/33

DrRacket  
○○○○

Values, expressions, & functions  
○○●○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

## > Canonical substitutions

Two rules:

- Functions are applied to values
- When there is a choice of possible substitutions, always take the **leftmost** choice.

Now, for any expression:

- there is at most one choice of substitution;
- the computed final result is the same as for other choices.

PL Design  
○○○○  
13/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○●○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

## > The use of parentheses: function application

There are two uses of parentheses in our usual mathematical notation. We've just seen one of them: function application.

The parentheses identify the arguments the function is applied to.

$f(3)$

$g(1, 2)$

PL Design  
○○○○  
14/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○●○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

## > The use of parentheses: ordering

- In arithmetic expressions, we often place operators between their operands.
- Example:  $3 - 2 + 4/5$ .
- We have some rules (division before addition, left to right) to specify order of operation.
- Sometimes these do not suffice, and parentheses are required.
- Example:  $(6 - 4)/(5 + 7)$ .



<https://www.xkcd.com/992/>

PL Design  
○○○  
15/33

DrRacket  
○○○

Values, expressions, & functions  
○○○○○●○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○

Programming  
○○○○  
CS 135

## > The use of parentheses: harmonization

If we treat **infix operators** (+, −, etc.) like functions, we don't need parentheses to specify order of operations:

Example:  $3 - 2$  becomes  $-(3, 2)$

Example:  $(6 - 4)/(5 + 7)$  becomes  $/(-(6, 4), +(5, 7))$

The substitution rules we developed for functions now work uniformly for functions and operators.

Parentheses now have only one use: function application.

PL Design  
○○○  
16/33

DrRacket  
○○○

Values, expressions, & functions  
○○○○○●○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○

Programming  
○○○○  
CS 135

## > Function application in Racket

Racket writes its functions slightly differently: the function name moves *inside* the parentheses, and the commas are changed to spaces.

Example:  $g(1, 3)$  becomes  $(g 1 3)$

Example:  $(6 - 4)/(5 + 7)$  becomes  $(/ (- 6 4) (+ 5 7))$

These are valid Racket expressions (once  $g$  is defined).

Functions and mathematical operations are treated exactly the same way in Racket.

PL Design  
○○○  
17/33

DrRacket  
○○○

Values, expressions, & functions  
○○○○○●○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○

Programming  
○○○○  
CS 135

## > Expressions in Racket

$3 - 2 + 4/5$  becomes `(+ (- 3 2) (/ 4 5))`

$(2 + 4/5 + 1)/3$  becomes `(/ (+ (+ 2 (/ 4 5)) 1) 3)`

Extra parentheses are harmless in arithmetic expressions.

They are harmful in Racket.

Only use parentheses when necessary (to signal a function application or some other Racket syntax).

PL Design  
○○○○  
18/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○●○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

### Exercise 2

Guess how to translate each expression into Racket. Enter them in DrRacket's interactions frame to check your work.

$$2 + 3$$

$$2 \times 3$$

$$44 - 2$$

### Exercise 3

Guess how to translate each expression into Racket. Enter them in DrRacket's interactions frame to check your work.

$$3 \times 4 + 2$$

$$\frac{2 + 4}{5 - 1}$$

$$3(1 + (6/2 + 5))$$

## > Evaluating a Racket expression

We use a process of substitution, just as with our mathematical expressions.

Each step is indicated using the 'yields' symbol  $\Rightarrow$ .

$( * (- 6 4) (+ 3 2) ) \Rightarrow$

$( * 2 (+ 3 2) ) \Rightarrow$

$( * 2 5 ) \Rightarrow$

10

## > Expressions in Racket

Racket has many built-in functions which can be used in expressions:

- Arithmetic operators: `+`, `-`, `*`, `/`
- Constants: `e`, `pi`
- Functions: `(abs x)`, `(max x y ...)`, `(ceiling x)` `(expt x y)`, `(exp x)`, ...

Look in DrRacket's "Help Desk". The web page that opens has many sections. The most helpful is under **Teaching**, then "*How to Design Programs Languages*", section 1.5.

## > Racket expressions causing errors

What is wrong with each of the following?

- `(5 * 14)`
- `(* (5) 3)`
- `(+ (* 2 4)`
- `(* + 3 5 2)`
- `(/ 25 0)`

**Syntax error:** An error discovered when reading an expression.

**Run-time error:** An error discovered when evaluating an expression.

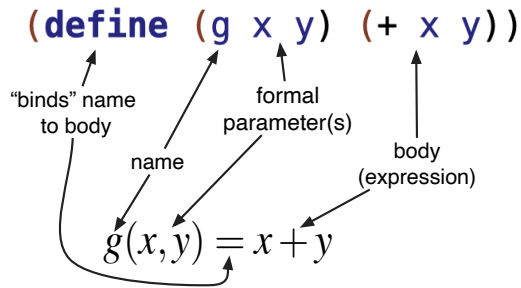


## Defining functions

A function definition consists of:

- a **name** for the function,
- a list of **parameters**,
- a single **body** expression.

(Racket definition on top; math on the bottom.)



The body expression typically uses the parameters together with other built-in and user-defined functions.

PL Design  
○○○○  
22/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
●○○○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

## Defining functions

Our definitions  $f(x) = x^2$ ,  $g(x, y) = x + y$  become

```
(define (f x) (sqr x))  
(define (g x y) (+ x y))
```

**define** is a **special form** (looks like a Racket function, but not all of its arguments are evaluated).

It **binds** a name to an expression (which uses the parameters that follow the name).

PL Design  
○○○○  
23/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
●○○○○○○○○○○○○○○

Programming  
○○○○○  
CS 135

## Exercise 4

In DrRacket's definitions frame (the top one), use **define** to create a function (`add-twice a b`) that calculates  $a + 2b$ .

Add an expression such as  
(`add-twice 3 5`)

Click the “Run” button and verify that DrRacket prints the correct answer in the interactions pane (13 for the expression give above).

Create and try out at least two other expressions that use `add-twice`.

## > Applying user-defined functions in Racket

An application of a user-defined function substitutes arguments for the corresponding parameters throughout the definition's expression.

```
(define (g x y) (+ x y))
```

The substitution for `(g 3 5)` would be `(+ 3 5)`.

## Exercise 5

Given these definitions:

```
(define (foo x) (+ x 4))  
(define (bar a b) (+ a a b))
```

What is the value of this expression? `(* (foo 0) (bar 5 (/ 8 (foo 0))))`

Try to figure it out by hand, then compare to the result calculated by DrRacket.

## > Applying user-defined functions in Racket

When faced with choices of substitutions, we use the same rules defined earlier:

- 1 apply functions only when all arguments are simple values
- 2 when you have a choice, take the leftmost one

<pre>(g (g 1 3) (f 3)) ⇒ (g (+ 1 3) (f 3)) ⇒ (g 4 (f 3)) ⇒ (g 4 (sqr 3)) ⇒ (g 4 9) ⇒ (+ 4 9) ⇒ 13</pre>	<pre>g(g(1, 3), f(3)) = g(1 + 3, f(3)) = g(4, f(3)) = g(4, 3<sup>2</sup>) = g(4, 9) = 4 + 9 = 13</pre>
---	--

## Parameters

Parameter names have meaning only within the body of the function.

Implications:

- The use of  $x$  in  $f$  and  $g$  are independent of each other.

```
(define (f x y) (+ x y))
(define (g x z) (* x z))
```

- The following two function definitions define the same function because the only difference is in the parameter names.

```
(define (f x y) (+ x y))
(define (f a b) (+ a b))
```

PL Design  
○○○○  
26/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○●○○○○○○

Programming  
○○○○○  
CS 135

## Defining constants

The definitions  $k = 3$ ,  $p = k^2$  become

```
(define k 3)
(define p (sqr k))
```

The effect of `(define k 3)` is to bind the name  $k$  to the value 3.

In `(define p (sqr k))`, the expression `(sqr k)` is first evaluated to give 9, and then  $p$  is bound to that value.

PL Design  
○○○○  
27/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○●○○○○○○

Programming  
○○○○○  
CS 135

## > Advantages of constants

- Can give meaningful names to useful values (e.g. `interest-rate`, `passing-grade`, and `starting-salary`).
- Reduces typing and errors when such values need to be changed
- Makes programs easier to understand

Notes:

- Constants can be used in any expression, including the body of function definitions
- Sometimes (incorrectly) called variables, but their values cannot be changed (until CS 136)

PL Design  
○○○○  
28/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○●○○○○○○

Programming  
○○○○○  
CS 135

## Exercise 6

Given the definitions, try to determine the value of each expression.

Check your understanding by comparing to what DrRacket gives.

```
1 (define x 4)
  (define (f x) (* x x))
  (f 3) ; => ?
```

```
2 (define (huh? huh?) (+ huh? 2))
  (huh? 7) ; => ?
```

```
3 (define y 3)
  (define (g x) (+ x y))
  (g 5) ; => ?
```

## Exercise 7

Try out the following lines of code in the definitions pane. If you change the order of the first two lines, what happens and why?

```
(define x (+ 2 3))
(define y (+ x 4.5))
x
y
```

## Scope

The **scope** of an identifier is where it has effect within the program.

```
(define x 3)
```

```
(define (f x y)
  (- x y))
```

```
(define (g a b)
  (+ a b x))
```

```
(+ (f 2 x) 1)
```

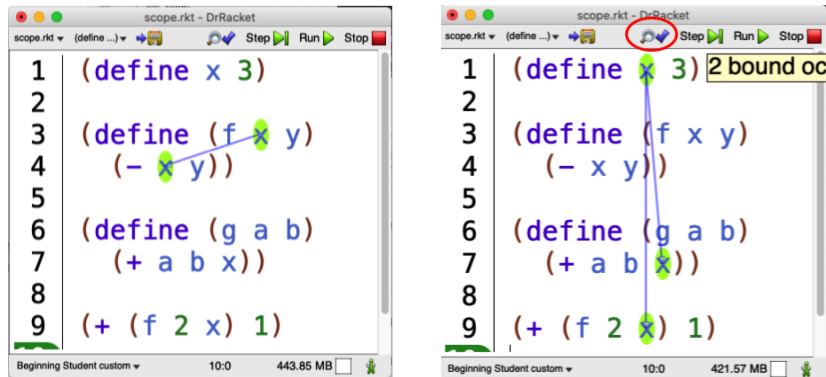
```
(define x 3)
```

- Two kinds of scope (for now): global and function
- The smallest enclosing scope has priority
- Duplicate identifiers within the same scope will cause an error

```
(define f 3)
(define (f x) (sqr x))
Racket Error: f: this name
was defined...
```

## Scoping tools in DrRacket

DrRacket can help you identify an identifier's scope.



## Programming in DrRacket

Use the definitions window:

- Can save and restore your work to/from a file
- Can accumulate definitions and expressions
- Run button loads contents into Interactions window
- Provides a Stepper to let one evaluate expressions step-by-step
- Features: error highlighting, subexpression highlighting, syntax checking

## > Programs in Racket

A Racket program is a sequence of definitions and expressions.

The expressions are evaluated, using substitution, to produce values.

Expressions may also make use of **special forms** (e.g. **define**), which look like functions, but don't necessarily evaluate all their arguments.

## Goals of this module

- You should understand the basic syntax of Racket, how to form expressions properly, and what DrRacket might do when given an expression causing an error.
- You should be comfortable with these terms: function, parameter, application, argument, constant, expression.
- You should be able to define and use simple arithmetic functions.
- You should understand the purposes and uses of the Definitions and Interactions windows in DrRacket.

PL Design  
○○○○  
33/33

DrRacket  
○○○○

Values, expressions, & functions  
○○○○○○○○○○○○○○○○  
02: Functions

Defining functions  
○○○○○○○○○○○○○○

Programming  
○○●○○  
CS 135

## Exercise 8

Write a Racket function corresponding to

$$g(x, y) = x\sqrt{x} + y^2$$

((`sqrt n`) computes  $\sqrt{n}$  and (`sqr n`) computes  $n^2$ .)

## Exercise 9

Evaluate the following program manually to determine what the result should be.

Then run it in Racket to check your work:

Note: (`sqrt n`) computes  $\sqrt{n}$  and (`sqr n`) computes  $n^2$ .

```
(define (disc a b c) (sqrt (- (sqr b) (* 4 (* a c)))))  
(define (proot a b c) (/ (+ (- 0 b) (disc a b c)) (* 2 a)))  
(proot 1 3 2) ; => ?
```