Tutorial 9

The goal of this tutorial is to reinforce the following material:

- Efficiency
Efficiency

• When looking at a function, it is often useful to understand its running time.

• To do this, we compute the running time as a function of the input size, and use Big O notation to simplify the computation.

In this course, you will see the following running times:

\[ O(1) \quad O(\log n) \quad O(n) \quad O(n \log n) \quad O(n^2) \quad O(n^3) \quad O(2^n) \]
Recursive Functions

Recall the steps for a recursive function:

1. Identify the order of the function *excluding* any recursion

2. Determine the size of the input for the next recursive call(s)

3. Write the full *recurrence relation* (combine step 1 & 2)

4. Look up the closed-form solution in a table
## Recurrence Relations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(n) = O(1) + T(n - k_1) )</td>
<td>( = O(n) )</td>
</tr>
<tr>
<td>( T(n) = O(n) + T(n - k_1) )</td>
<td>( = O(n^2) )</td>
</tr>
<tr>
<td>( T(n) = O(n^2) + T(n - k_1) )</td>
<td>( = O(n^3) )</td>
</tr>
<tr>
<td>( T(n) = O(1) + T(\frac{n}{k_2}) )</td>
<td>( = O(\log n) )</td>
</tr>
<tr>
<td>( T(n) = O(1) + k_2 \cdot T(\frac{n}{k_2}) )</td>
<td>( = O(n) )</td>
</tr>
<tr>
<td>( T(n) = O(n) + k_2 \cdot T(\frac{n}{k_2}) )</td>
<td>( = O(n \log n) )</td>
</tr>
<tr>
<td>( T(n) = O(1) + T(n - k_1) + T(n - k'_1) )</td>
<td>( = O(2^n) )</td>
</tr>
</tbody>
</table>

where \( k_1, k'_1 \geq 1 \) and \( k_2 > 1 \)
Iterative analysis

Recall the steps for an iterative function:

1. Work from the *innermost* loop to the *outermost*

2. Determine the number of iterations in the loop (in the worst case) in relation to the size of the input \( n \) or an outer loop counter

3. Determine the running time per iteration

4. Write the summation(s) and simplify the expression
Common Summations

\[ \log n \sum_{i=1}^{n} O(1) = O(\log n) \]

\[ n \sum_{i=1}^{n} O(1) = O(n) \]

\[ n \sum_{i=1}^{n} O(n) = O(n^2) \]

\[ n \sum_{i=1}^{n} O(i) = O(n^2) \]

\[ n \sum_{i=1}^{n} O(i^2) = O(n^3) \]