Goal of this Tutorial

The goal of this tutorial is to reinforce the following material:

- Efficiency
Efficiency

- When looking at a function, it is often useful to understand its running time.
- To do this, we compute the running time as a function of the input size, and use Big O notation to simplify the computation.

In this course, we will see the following running times:

\[ O(1) \quad O(\log n) \quad O(n) \quad O(n \log n) \quad O(n^2) \quad O(n^3) \quad O(2^n) \]

These are similar to what you may have seen in Calculus 2 on growth rates of sequences.
Recursive Functions

Recall the steps for a recursive function:

1. Identify the order of the function excluding any recursion

2. Determine the size of the input for the next recursive call(s)

3. Write the full recurrence relation (combine step 1 & 2)

4. Look up the closed-form solution in a table
### Recurrence Relations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(n) = O(1) + T(n - k_1)$</td>
<td>$= O(n)$</td>
</tr>
<tr>
<td>$T(n) = O(n) + T(n - k_1)$</td>
<td>$= O(n^2)$</td>
</tr>
<tr>
<td>$T(n) = O(n^2) + T(n - k_1)$</td>
<td>$= O(n^3)$</td>
</tr>
<tr>
<td>$T(n) = O(1) + T\left(\frac{n}{k_2}\right)$</td>
<td>$= O(\log n)$</td>
</tr>
<tr>
<td>$T(n) = O(1) + k_2 \cdot T\left(\frac{n}{k_2}\right)$</td>
<td>$= O(n)$</td>
</tr>
<tr>
<td>$T(n) = O(n) + k_2 \cdot T\left(\frac{n}{k_2}\right)$</td>
<td>$= O(n \log n)$</td>
</tr>
<tr>
<td>$T(n) = O(1) + T(n - k_1) + T(n - k'_1)$</td>
<td>$= O(2^n)$</td>
</tr>
</tbody>
</table>

where $k_1, k'_1 \geq 1$ and $k_2 > 1$
Iterative analysis

Recall the steps for an iterative function:

1. Work from the *innermost* loop to the *outermost*

2. Determine the number of iterations in the loop (in the worst case) in relation to the size of the input ($n$) or an outer loop counter

3. Determine the running time per iteration

4. Write the summation(s) and simplify the expression
Common Summations

\[
\sum_{i=1}^{\log n} O(1) = O(\log n)
\]

\[
\sum_{i=1}^{n} O(1) = O(n)
\]

\[
\sum_{i=1}^{n} O(n) = O(n^2)
\]

\[
\sum_{i=1}^{n} O(i) = O(n^2)
\]

\[
\sum_{i=1}^{n} O(i^2) = O(n^3)
\]
Exercise: Magic Square

- Go into Seashell and implement our modified version of the magic square puzzle. This exercise should give you practice in writing a backtracking algorithm. For this program, you may assume that all empty squares contain the value 0.

- `solve_square(magic, target)` will solve the given 3x3 magic square such that the sum of every row and column is target and all of the entries are positive.

  - requires: magic is a valid square
  - effects: modifies magic