1. [2 marks] Calculate the following sums. The answers should be in the same base as the original numbers. You may use any technique you like to calculate the addition. It is not necessary to show your work. Note that calculators will not be allowed on the midterm.
   a) \(0x8AB5 + 0x3787\) (Note the prefix 0x for a number indicates hexadecimal)
   b) \(10100101_2 + 11001100_2\) (these are binary numbers)

**Solutions**
   a) \(0x8AB5 + 0x3787 = 0xC23C\)
   b) \(10100101_2 + 11001100_2 = 101110001_2\)

**Marking Notes**
   - Each part is worth 1 mark – all or nothing. No work needs to be shown.
2. **[2 marks]** Calculate the following differences. Use twos complement notation assuming 8-bit numbers and show your work. All computation must be done with binary numbers. Indicate if there is an overflow for either of the calculations.
   a) -75+59

*Solutions*

   a) 75 = 01001011
      -75 = 10110100+1 = 10110101₂
      59 = 00111011₂

      -75 + 59 = 10110101₂ + 00111011₂ = 11110000₂
      11110000₂ = -16

*Marking Notes*

- 1 mark – properly converting -75 to its twos complement version
- 1 mark – correct answer: -16 (no marks if binary sum correct, but not the conversion to decimal)
- Showing the conversion from decimal to binary is not necessary. Show the conversion from binary to decimal.
3. **[2 marks]** Using Boolean algebra, prove that the following two Boolean expressions are equivalent. Show all steps without skipping any. (Dot (.) represents AND). (Feel free to use the Boolean algebra rules in the lecture slides.)

\[(\neg (A + B)) + (\neg A \cdot (B+C)) = \neg A\]

**Possible Solution**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Rule(s) Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\neg (A + B)) + (\neg A \cdot (B+C)))</td>
<td>Original Expression</td>
</tr>
<tr>
<td>((\neg A \cdot \neg B) + (\neg A \cdot (B+C)))</td>
<td>De Morgan’s Law.</td>
</tr>
<tr>
<td>((\neg A \cdot \neg B) + (\neg A \cdot B) + (\neg A \cdot C))</td>
<td>Distributive.</td>
</tr>
<tr>
<td>(\neg A \cdot (\neg B + B + C))</td>
<td>Distributive.</td>
</tr>
<tr>
<td>(\neg A \cdot (1 + C))</td>
<td>Complement.</td>
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<tr>
<td>(\neg A)</td>
<td>Annihilator.</td>
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</table>

The same result may also be reached via another route, but needs to be done in a step by step manner.

**Marking Notes**
- The required rule does not need to be stated.
- However, if a step is skipped, without any explanation, then -0.5
- +0.5 for each correct step (the two distributive steps are 0.25 each, but round off the final marks to the upper .5 value)
- Providing truth tables is not a proof
4. **[3 marks]**

Draw a logic circuit (circuit with logic gates) for \((\neg(A + B)).(C + D).(-C)\). Do not simplify the expression. Draw the gates clearly.

**Solutions**

\[
\begin{align*}
\text{A} & \quad \text{B} \\
\text{C} & \quad \text{D}
\end{align*}
\]

\((\neg(A + B)).(C + D).(-C) = \textbf{(NOT (A OR B)) AND (C OR D) AND (NOT C)}\)

There are six operations above (in bold). 0.5 mark for getting each operation and its operands right. Instead of NOR as above, students may use NOT, OR. Instead of 3-input AND, they may use two 2-input AND gates, so such variations are ok. If an incorrect/confusing gate is drawn, or if the operands are incorrect for an operation, no marks for that.
5. [3 marks]

a) Consider the following truth table:

<table>
<thead>
<tr>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

Write a Boolean expression that produces the result of this truth table. Then, simplify the expression such that \( W, X, Y, Z \) appear exactly once in the resulting expression. Show all steps.

**Solution:**

\[
\neg W \neg XYZ + \neg WX \neg YZ + \neg WXYZ \quad \text{(1.5 marks). No partial marking. The ordering of terms can be different, (and the students can choose different ways to represent not, and, or, or symbols) but this expression needs to be the same.}
\]

Simplifying:

\[
\neg W \neg (\neg XY + X \neg Y + XY) \quad \text{(0.5 marks)}
\]
\[
\neg W \neg (\neg XY + X (\neg Y + Y)) \quad \text{(0.5 marks)}
\]
\[
\neg WZ (\neg X + X) \quad \text{(0.5 marks)}
\]
\[
\neg WZ (\neg X + X)(Y + X) \quad \text{(0.5 marks)}
\]
\[
\neg WZ(X + Y) \quad \text{(0.5 marks)}
\]

[The answer needs to be the above expression (Something like \( \neg WZX + \neg WZY \) does not satisfy the appear exactly once criteria.) As long as the answer is correct and it shows any three intermediary steps correctly, 1.5 marks.]