Cache CPI and DFAs and NFAs

CS230 Tutorial 10
Multi-Level Cache: Calculating CPI

When a memory access is attempted, what are the possible results?
Multi-Level Cache: Calculating CPI

We have three possibilities, find the formula for each:

- **L1 Cache Hit Cost**
  - This is considered part of normal execution. CPI cost is zero.

- **L2 Cache Hit Cost**
  - We missed L1 cache but found the data in the L2 cache
  - This is $L1$-miss-penalty $\times$ $L1$-miss-chance

- **L2 Cache Miss Cost**
  - We missed L1 and L2 cache, have to go all the way to memory
  - This is main-memory-access-time $\times$ global-cache-miss-chance
    - global-cache-miss-chance could be calculated as $L1$-miss-chance $\times$ L2-miss-chance

- **CPI Formula is:** base-CPI + L2-cache-hit-CPI + L2-cache-miss-CPI
Multi-Level Cache: CPI Example

Consider a processor running at 5GHz with a Base-CPI of 2.4. It has a two-level cache. The L1 miss penalty is 2ns. The main memory access time is 60ns.

The L1 cache hit rate is 85%. The L2 cache hit rate is 90%.

Calculate the total CPI of this processor with and without an L2 cache. What is the speedup gained by having an L2 cache?
Multi-Level Cache: CPI Example Solution

A 5GHz processor has a cycle time of 0.2ns.
L2 cache hit CPI = 2ns * 0.15 = 0.3ns = 1.5 CPI
L2 cache miss CPI = 60ns * 0.15 * 0.10 = 0.9ns = 4.5 CPI
So the effective CPI of this processor is: 2.4 + 1.5 + 4.5 = 8.4 CPI

If there was no L2 cache this would be:

L1 cache miss = 60ns * 0.15 = 9ns = 45 CPI
So the effective CPI of this processor would have been: 2.4 + 45 = 47.4 CPI

The speedup is therefore: 47.4/8.4 = ~5.6 times faster!
Multi-Level Cache: CPI Practice

Consider a processor running at 2GHz with a Base-CPI of 1.4. It has a two-level cache. The L1 miss penalty is 10ns. The main memory access time is 40ns.

The L1 cache hit rate is 90%. The global cache miss rate with an L2 cache is 2%.

Calculate the total CPI of this processor with and without an L2 cache. What is the speedup gained by having an L2 cache?
Multi-Level Cache: CPI Practice Solution

A 2GHz processor has a cycle time of 500ps.
L2 cache hit CPI = 10ns * 0.1 = 1ns = 1000ps = 2 CPI
L2 cache miss CPI = 40ns * 0.02 = 0.8ns = 800ps = 1.6 CPI
So the effective CPI of this processor is: 1.4 + 2 + 1.6 = 5 CPI

If there was no L2 cache this would be:
L1 cache miss = 40ns * 0.1 = 4ns = 4000ps = 8 CPI
So the effective CPI of this processor would have been: 1.4 + 8 = 9.4 CPI

The speedup is therefore: 9.4/5 = 1.88 times faster!
Regular Languages

- There are various types of *Formal Languages*.
- Right now we care about *Regular Languages* a subset of formal languages.
- Regular languages can be represented in various ways:
  - DFAs
  - NFAs
  - Regular Expressions (not covered yet)
Deterministic Finite Automata (DFAs)

DFAs are a graph (in the discrete math sense) where there is a set of states and transitions between those states that occur upon consuming a character from the input.

The idea is that you can take a string (i.e. a list of characters) and check if it is “accepted” by (i.e. fits in) the language by seeing if you can follow along the flow of the DFA and reach the accept state (denoted by a double circle) at the end of the string.

If you make it to an accept state at the end of the string, then the string is in the regular language defined by that DFA.

If you are not in an accept state at the end of the string, then the string is not in the regular language defined by that DFA. It is possible to enter an accept state and then leave it again. The string must end in an accept state to be accepted.
DFA Examples

- Draw a DFA for the language of any combination of the letters a and b.
- Draw a DFA for the language of at least one a, following by zero or more numeric digits followed by and x and then an optional y.
DFA Example Solution 1

Draw a DFA for the language of any combination of the letters a and b.

Solution:

\[
\begin{array}{c}
q_0 \\
\end{array}
\]

\[
\begin{array}{c}
\text{start} \\
a, b
\end{array}
\]
DFA Example Solution 2

Draw a DFA for the language represented by at least one a, following by zero or more numeric digits followed by and x and then an optional y.

Solution:
DFA Practice

- Draw a DFA for the language of any number of a’s, followed by an even number of c’s.
- Draw a DFA for the language of two or three b’s followed by zero or more c’s followed by an a or d.
- Draw a DFA for the language of at least two a’s followed by an odd number of d’s followed by two a’s or one c.
DFA Practice Solution 1

Draw a DFA for the language of any number of a’s, followed by an even number of c’s.

Solution:

![DFA Diagram](image)
DFA Practice Solution 2

Draw a DFA for the language of two or three b’s followed by zero or more c’s followed by an a or d.

Solution:
DFA Practice Solution 3

Draw a DFA for the language of at least two a’s followed by an odd number of d’s followed by two a’s or one c.

Solution:
Non-Deterministic Finite Automata (NFAs)

With DFAs, there is no ambiguity at all, and only one possible path between states for a given string. With NFAs, there are situations where there could be more than one choice for a move from state to state while parsing a string.

NFAs can have (which DFAs do not have):

- epsilon transitions (state transitions on the empty character)
- multiple transitions on the same state for a particular character (for example, on one state, there could be 2 arrows out on the same character)

NFAs can always be translated to an equivalent DFA.
NFA Example

Let’s look at one of the examples we’ve been looking at so far:

Draw a NFA for the language of at least one a, following by zero or more numeric digits followed by and x and then an optional y.

Recall that the DFA is as follows:
NFA Example Solution

Example: Draw a NFA for the language of at least one $a$, following by zero or more numeric digits followed by and $x$ and then an optional $y$.

Solution:

Note: there are more than one possible NFA for this problem, and I am purposely showing many NFA features and not going for optimal NFA construction.
NFA Practice

- Draw an NFA for the language of any number of a’s, followed by any number of repetitions of abba.
- Draw an NFA for the language of two or three b’s followed by any number of c’s followed by an a or d.
- Draw an NFA for the language of one or more d’s followed by any number of c’s followed by cd, or ac, or nothing.
NFA Practice Solution 1

Draw an NFA for the language of any number of a’s, followed by any number of repetitions of abba.

Solution:
NFA Practice Solution 2

Draw an NFA for the language of two or three b’s followed by zero or more c’s followed by an a or d.

Solution:
Draw an NFA for the language of one or more d’s followed by any number of c’s followed by cd, or ac, or nothing.

Solution:
Assignment reminders

- Submit a `.txt` XOR a `.pdf` for each question
  - Do not submit both for the same question!
  - You may submit a `.pdf` for one question and a `.txt` for a different question

- Make sure your diagrams and tables are clear and easy to read
  - Make sure to leave enough space