Problems

Module 1

Finding a Tour

Input: A complete graph with a weight on each edge
Output: A cycle that includes all the vertices (or a tour)

Tour Decision

Input: A complete graph with a weight on each edge, and a maximum budget B
Output: Yes or no, answering “Is there a tour of weight at most B?”

Tour Search

Input: A complete graph with a weight on each edge, and a maximum budget B
Output: A tour of weight at most B

Tour Counting

Input: A complete graph with a weight on each edge, and a maximum budget B
Output: The number of tours of weight at most B

Tour Enumeration

Input: A complete graph with a weight on each edge, and a maximum budget B
Output: The set of all tours of weight at most B

Tour Constructive Optimization

Input: A complete graph with a weight on each edge
Output: A tour of minimum weight

Tour Evaluation Optimization

Input: A complete graph with a weight on each edge
Output: The minimum weight of a tour

Finding distinct items

Input: A set of numbers
Output: The numbers in the set that appear exactly once
Travelling Salesperson Problem (TSP)

Input: A complete graph with a weight on each edge

Output: A tour of minimum weight

Pattern matching in a grid

Input: A pattern grid and a target grid

Output: Yes or no, answering “Does the pattern appear in the target?”

Module 2 Number 1 odd

Input: A sequence of numbers

Output: Yes or no, answering “Is 1 odd?”

First three odd

Input: A sequence of numbers of length at least three

Output: Yes or no, answering “Are the first three numbers all odd?”

All odd

Input: A nonempty sequence of numbers

Output: Yes or no, answering “Are all the numbers odd?”

Any odd

Input: A nonempty sequence of numbers

Output: An odd number in the sequence, or “None” if none exists

Maximum

Input: A nonempty sequence of numbers

Output: A number in the sequence with maximum value

Module 3

Scheduling activities

Input: A set of $n$ activities, where activity $i$ has start time $s_i$ and finish time $f_i$

Output: A schedule that has the maximum number of nonoverlapping activities, that is, for each pair chosen $f_i \leq s_j$ or $f_j \leq s_i$

Making change

Input: A set of integer coin values $c_1, c_2, \ldots, c_k$ and an integer $x$
**Output:** Numbers of coins of each denomination such that the total number of coins is the smallest possible and the total value is $x$

**Sorting**

**Input:** A set of numbers

**Output:** The numbers in order from smallest to largest

**Fractional knapsack**

**Input:** A set of $n$ types of objects, where object $i$ has integer weight $w_i$ and integer value $v_i$, and an integer weight bound $W$

**Output:** A list of fractions $0 \leq x_i \leq 1$ such that $\sum_{i=1}^{n} x_i \cdot w_i \leq W$ and $\sum_{i=1}^{n} x_i \cdot v_i$ is maximized

**Single source cheapest paths**

**Input:** A graph $G$ with non-negative edge weights and a source vertex $s \in V(G)$

**Output:** The least-cost paths from $s$ to each vertex in $G$, where the cost is the sum of the weights of edges in the path

**Minimum spanning tree**

**Input:** A connected graph $G$ with non-negative edge weights

**Output:** A subset $A$ of $E(G)$ that forms a tree from all of $V(G)$ and the sum of the weights of edges in $A$ is as small as possible

$k$-Colouring

**Input:** A graph $G$ and an integer $k$

**Output:** Yes or no, answering “Is $G$ is $k$-colourable?”

3-Colouring

**Input:** A graph $G$

**Output:** Yes or no, answering “Is $G$ is 3-colourable?”

**Module 4 Questions**

**Input:** A range of integers from $a$ to $b$, $a \leq b$, and an integer $k$

**Output:** The only number in the range that is consistent with the answers to the $k$ questions.

**Maximum subtotal**

**Input:** A sequence of $n$ numbers $a_1$ through $a_n$
Output: $\sum_{k=i}^{j} a_k$ that is the largest possible

Selection

Input: A set of $n$ numbers and a value $j$, $1 \leq j \leq n$.
Output: A number in the set that is the $j$th largest.

Closest pair of points

Input: A collection of $n$ points, where point $p_i = (x_i, y_i)$
Output: Points $p_a$ and $p_b$ in the collection, $a \neq b$, such that $\text{dist}(p_a, p_b)$ is minimized.

Matrix multiplication

Input: Two $n \times n$ matrices $A$ and $B$
Output: The product of $A$ and $B$

Module 5

Matrix-chain multiplication

Input: A sequence (chain) of matrices $M_0, \ldots, M_{n-1}$, where $M_i$ has dimension $d_i \times d_{i+1}$
Output: An order of multiplications that results in the smallest number of multiplications of pairs of values

Longest common subsequence

Input: A string $X$ of length $m$ and a string $Y$ of length $n$
Output: A string $Z$ that is a subsequence of both $X$ and $Y$ and of maximum length

All-pairs cheapest paths

Input: A graph $G$ with non-negative edge weights
Output: The least-cost paths between each pair of vertices in $G$

Knapsack

Input: A set of $n$ types of objects, where object $i$ has integer weight $w_i$ and integer value $v_i$, and an integer weight bound $W$
Output: A subset of objects with total weight at most $W$ and the maximum total value

Module 6

Any odd

Input: A nonempty sequence of numbers
Output: Yes or no, answering “Does the sequence contain an odd number?”
**Shortest Path Search**

**Input:** A graph $G$ and vertices $u$ and $v$

**Output:** A shortest path from $u$ to $v$

**Shortest Path Decision**

**Input:** A graph $G$ and vertices $u$ and $v$, and an integer $k$

**Output:** Yes or no, answering “Is there a path from $u$ to $v$ of length at most $k$?”

**3-Colouring**

**Input:** A graph $G$

**Output:** Yes or no, answering “Is $G$ 3-colourable?”

**Composite**

**Input:** An integer $N$

**Output:** Yes or no, answering “Is $N$ composite?”

**Hamiltonian Cycle**

**Input:** A graph $G$

**Output:** Yes or no, answering “Does $G$ have a Hamiltonian cycle?”

**Vertex Cover**

**Input:** A graph $G$ and an integer $k$

**Output:** Yes or no, answering “Does $G$ have a vertex cover of size at most $k$?”

**Independent Set**

**Input:** A graph $G$ and an integer $k$

**Output:** Yes or no, answering “Does $G$ have an independent set of size at least $k$?”

**Clique**

**Input:** A graph $G$ and an integer $k$

**Output:** Yes or no, answering “Does $G$ have a clique of size at least $k$?”

**Module 7**

**Integer Knapsack**

**Input:** A set of $n$ types of objects, where object $i$ has weight $w_i$ and value $v_i$, and a weight bound $W$
Output: A list of integers $c_1, \ldots, c_n$ such that $\sum_{i=1}^{n} c_i \cdot w_i \leq W$ and $\sum_{i=1}^{n} c_i \cdot v_i$ is maximized

Partition

Input: A group of $n$ objects, where object $i$ has weight $w_i$

Output: A partition of the objects into groups $A$ and $B$ with minimum difference in the sum of the weights of each part

Module 8

Job Scheduling

Input: A set of jobs where job $j$ takes time $t(j)$

Output: An assignment of jobs to $m$ machines such that the makespan is minimized.

Bin Packing

Input: A group of $n$ objects, where object $i$ has size $s_i \leq 1$

Output: An assignment of objects to bins, where the sum of the sizes of the objects in a bin is at most 1, and the total number of bins is as small as possible