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Case study

Problem: Given Racket code, are all pairs of parentheses and brackets matched correctly?

Example:

```
(define (is-odd? x)
  (cond [(= x 0) false]
        [else (is-even? (sub1 x))])
```

Recipe for user/plan

1. Determine types of data and operations.
2. For each type, choose/modify/create an ADT.
3. Develop pseudocode algorithm using ADT operations.
4. Calculate cost of algorithm with respect to costs of operations.
5. Using information from provider, choose best option.
ADT Stack

The ADT Stack supports “first in last out” arrangement of data.
Preconditions: For all S is a stack and Data any data item; for TOP and POP, S is not empty.
Postconditions: Mutation by PUSH (adds Data to the top of the stack) and POP (deletes data item from the top of the stack).

<table>
<thead>
<tr>
<th>Name</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>CREATE()</td>
<td>a new empty stack</td>
</tr>
<tr>
<td>IS_EMPTY(S)</td>
<td>True if empty, else False</td>
</tr>
<tr>
<td>TOP(S)</td>
<td>data item that is on top</td>
</tr>
<tr>
<td>PUSH(S, Data)</td>
<td></td>
</tr>
<tr>
<td>POP(S)</td>
<td>data item that was on top</td>
</tr>
</tbody>
</table>
Example of use of ADT Stack operations

Colours ← CREATE()

**print**(!IS EMPTY(Colours))

**Push**(Colours, "puce")

**Push**(Colours, "mauve")

One ← **Top**(Colours)

**Push**(Colours, "scarlet")

Two ← **Pop**(Colours)

**print**(One)

**print**(Two)
Case study pseudocode

1  Symbols ← CREATE()
2  for each character Char in the input string
3       if Char is ( or [ or {
4           PUSH(Symbols, Char)
5       if Char is ) or ] or }
6       if IS_EMPTY(Symbols)
7           return "Bad program"
8  Current ← POP(Symbols)
9       if Current is not the same type of symbol as Char
10           return "Bad program"
11       if IS_EMPTY(Symbols)
12           return "Good program"
13   else
14       return "Bad program"

Costs: CREATE C, IS_EMPTY E(k), PUSH A(k), POP D(k)
Step 5: Use information from provider

Before Step 5 can take place, the provider needs to work.

Choosing among implementations (provider/plan):

1. Create pseudocode of various options for data structures and algorithms to implement the ADT and its operations.
2. Analyze the costs of each operation for each implementation.
3. Provide options for packages of operation costs.
Contiguous implementation of ADT Stack

Data structures:
- Array of size s with all data items stored contiguously, starting at index 0
- Variable Top storing index of last slot storing a data item (or −1 if empty)

Algorithms:
- CREATE() does not initialize array; Top set to −1
- IS_EMPTY(S) checks if Top is −1
- TOP(S) returns item at index Top
- PUSH(S, Data) increments Top; stores Data at index Top
- POP(S) removes and returns data item at index Top; decrements Top
Analysis of contiguous implementation

- $CREATE()$: $\Theta(1)$
- $IS\_EMPTY(S)$: $\Theta(1)$
- $TOP(S)$: $\Theta(1)$
- $PUSH(S, Data)$: $\Theta(1)$
- $POP(S)$: $\Theta(1)$
Linked list implementation of ADT Stack

Data structures:

- Variable *Top* storing a pointer to a linked list of nodes, each storing a data item and a pointer to the next node in the list

Algorithms:

- *CREATE()* sets variable *Top* to null pointer
- *IS_EMPTY(S)* checks if *Top* is null
- *TOP(S)* returns the value in the node to which *Top* points
- *PUSH(S, Data)* creates a new node storing *Data* and with its *Next* pointer set to *Top*; updates *Top* to point to the new node
- *POP(S)* returns the value in the node to which *Top* points; updates *Top* to the *Next* pointer of the node to which it points
Analysis of linked implementation

- $CREATE()$: $\Theta(1)$
- $IS\_EMPTY(S)$: $\Theta(1)$
- $TOP(S)$: $\Theta(1)$
- $PUSH(S, Data)$: $\Theta(1)$
- $POP(S)$: $\Theta(1)$
ADT Queue

The ADT Queue supports “first in first out” arrangement of data. Preconditions: For all Q is a queue and Data any data item; for FIRST and DEQUEUE, Q is not empty.

Postconditions: Mutation by ENQUEUE (adds Data to the back of the queue) and DEQUEUE (deletes data item from the front of the queue).

<table>
<thead>
<tr>
<th>Name</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>CREATE()</td>
<td>a new empty queue</td>
</tr>
<tr>
<td>IS_EMPTY(Q)</td>
<td>True if empty, else False</td>
</tr>
<tr>
<td>FIRST(Q)</td>
<td>data item that is first</td>
</tr>
<tr>
<td>ENQUEUE(Q, Data)</td>
<td>data item that was first</td>
</tr>
</tbody>
</table>
Example of use of ADT Queue operations

Animals ← CREATE()
print(IS_EMPTY(Animals))
ENQUEUE(Animals, "dingo")
ENQUEUE(Animals, "echidna")
One ← DEQUEUE(Animals)
ENQUEUE(Animals, "wombat")
Two ← DEQUEUE(Animals)
print(One)
print(Two)
Data structure: Circular array

Contiguous:

- Array with entries
- *Head* stores index of the first item in the sequence
- *Tail* stores the last item stored in the sequence, or -1 if the sequence is empty

```
5
Head
2
Tail
```

```
d  e  f
0  1  2
```

```
a  b  c
4  5  6
```

```
3
4
```

```
7
```

```
Data structure: Circular linked list

Linked structure:
- Each linked node contains a data item and a *Next* pointer
- *Next* pointer of last node points to first node
- *Tail* pointer to last node

Note: Both the first and the last data items can be found in constant time.
Data structure: Doubly-linked list

Linked structure:

- Each linked node contains a data item, a Next pointer, and a Prev pointer
- *Head* pointer to first node
- Optional *Tail* pointer to last node
Data structure: Doubly-linked circular list

Linked structure:

- Each linked node contains a data item, a Next pointer, and a Prev pointer
- Next pointer of last node points to first node
- Prev pointer of first node points to last node
- Either one or both of Head pointer and Tail pointer are included

Note: Both the first and the last data items can be found in constant time.
ADTs with order imposed by operations

ADTs to consider:

- “First in last out” - ADT Stack
- “First in first out” - ADT Queue
- Specify a position in one dimension - ADT Indexed Sequence
- Specify a ranking - ADT Ranking
- Specify a position in two dimensions - ADT Grid
Mimicking behaviour of contiguous memory

Without specifying how the data is actually stored, create an ADT with the same features as an array:

- Specify the capacity of the ADT at the outset.
- Allow random access of data items by index.

Notes:

- Positions of data items depend on operations, not values
- Data items need not be orderable
ADT Indexed Sequence

The ADT Indexed Sequence allows access to data items by index.

Preconditions: For all $I$ is an indexed sequence, $Data$ is any data item, $Cap$ is a positive integer; and $Index$ is an integer, $0 \leq Index < Cap$.

Postconditions: Mutation by $ADD$ (adds/replaces item at $Index$) and $DELETE$ (deletes data item at $Index$, if any).

<table>
<thead>
<tr>
<th>Name</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CREATE(Cap)$</td>
<td>a new empty indexed sequence of capacity $Cap$ with all entries initialized to empty</td>
</tr>
<tr>
<td>$CAP(I)$</td>
<td>the capacity of $I$</td>
</tr>
<tr>
<td>$IS_EMPTY(I)$</td>
<td>$True$ if empty, else $False$</td>
</tr>
<tr>
<td>$LOOK_UP(I, Index)$</td>
<td>data item with index $Index$, if any, else $False$</td>
</tr>
<tr>
<td>$ADD(I, Index, Data)$</td>
<td></td>
</tr>
<tr>
<td>$DELETE(I, Index)$</td>
<td>data item with index $Index$, if any, else $False$</td>
</tr>
</tbody>
</table>

Note: We assume that $False$ is not a data item.
Example of use of ADT Indexed Sequence operations

```
Gifts ← CREATE(12)
print(CAP(Gifts))
ADD(Gifts, 2, "cows")
ADD(Gifts, 5, "rings")
ADD(Gifts, 7, "swans")
print(IS EMPTY(Gifts))
Discarded ← DELETE(Gifts, 5)
Fifth ← LOOK_UP(Gifts, 5)
ADD(Gifts, 2, "doves")
Second ← LOOK_UP(Gifts, 2)
print(Fifth)
print(Second)
```
Contiguous implementation 1

Idea: Have one array of indices and one of values.

Data structures:

- Array *Indices* of size s of indices (full slots first, any order)
- Array *Values* of size s of values (match order in *Indices*)
- Variable *Cap* storing s
- Variable *Last* containing index of last full entry or -1
Contiguous implementation 2

Idea: Store the index and value in an ADT Pair with operations Index and Value that return the two pieces of data.

Data structures:
- Array *Pairs* of size *s* storing ADT Pairs
- Variable *Cap* storing *s*
- Variable *Last* containing index of last full entry or -1
Contiguous implementation 3

Data structures:
- Array of size $s$; index in array matches position
- Variable $Cap$ storing $s$

Algorithms:
- $CREATE$ initializes all entries to empty; sets $Cap$ to $s$
- $CAP$ produces $Cap$
- $IS\_EMPTY$ checks entries until all checked or a value is found
- $LOOK\_UP$ checks position $Index$ in array
- $ADD$ updates value at position $Index$ in array
- $DELETE$ sets entry at position $Index$ to empty
Linked implementations of ADT Indexed Sequence

Ideas for linked lists (also consider other data structures):

1. s nodes, where the node in position \( \text{Index} \) stores the data item for \( \text{Index} \)
2. Each node stores an ADT Pair, any order
3. Each node stores an ADT Pair, ordered by index
4. Each node stores an ADT Pair, ordered by value
Observations

About implementations:

- An indexed sequence (ADT) is not an array (data structure).
- An indexed sequence does not have to be implemented as an array.
- A contiguous implementation of an indexed sequence does not have to be the obvious one.

About data structures:

- A data structure can consist of multiple arrays.
- An array can store more than one piece of information at an index.
- A linked list can store more than one piece of information in a linked node.
Cost of ADT Indexed Sequence $\text{LOOK\_UP}(l,\text{Index})$

As a function of $s$:  
- Contiguous 3, worst case $\Theta(1)$  
- Contiguous 3, best case $\Theta(1)$  
- Linked with $s$ nodes, worst case $\Theta(s)$ (last position)  
- Linked with $s$ nodes, best case $\Theta(1)$ (first position)

As a function of $s$ and $p$, the value of $\text{Index}$:  
- Contiguous 3, worst case $\Theta(1)$  
- Contiguous 3, best case $\Theta(1)$  
- Linked with $s$ nodes, worst case $\Theta(p)$  
- Linked with $s$ nodes, best case $\Theta(p)$
Mimicking behaviour of linked memory

Without specifying how the data is actually stored, create an ADT with the same features as a linked list:

- Each data item has a rank based on its position in the linked list.
- Adding or deleting an item changes the ranks of other items.
ADT Ranking

The ADT Ranking supports a ranking of data items (an assignment of distinct ranks from 0 to \( k - 1 \) for \( k \) data items).

Preconditions: For all \( R \) is a ranking, Data any data item; for \( \text{LOOK\_UP} \) and \( \text{DELETE} \) \( \text{Rank} \) is the rank of a data item in \( R \), and for \( \text{ADD} \) \( \text{Rank} \) can be the rank of a data item in \( R \) or one greater.

Postconditions: Mutation by \( \text{ADD} \) (adds Data with rank \( \text{Rank} \) and increments ranks of all other items with rank \( \text{Rank} \) or greater) and \( \text{DELETE} \) (deletes data item with rank \( \text{Rank} \) and decrements ranks of all items that had rank \( \text{Rank} + 1 \) or greater).

<table>
<thead>
<tr>
<th>Name</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( \text{CREATE}() )</td>
<td>a new empty ranking</td>
</tr>
<tr>
<td>( \text{IS_EMPTY}(R) )</td>
<td>( True ) if empty, else ( False )</td>
</tr>
<tr>
<td>( \text{MAX_RANKING}(R) )</td>
<td>maximum rank used ((-1) if empty)</td>
</tr>
<tr>
<td>( \text{LOOK_UP}(R, \text{Rank}) )</td>
<td>data item that has rank ( \text{Rank} )</td>
</tr>
<tr>
<td>( \text{ADD}(R, \text{Rank}, \text{Data}) )</td>
<td></td>
</tr>
<tr>
<td>( \text{DELETE}(R, \text{Rank}) )</td>
<td>data item that had rank ( \text{Rank} )</td>
</tr>
</tbody>
</table>
Example of use of ADT Ranking operations

\[
\begin{align*}
\text{Desserts} & \leftarrow \text{CREATE()} \\
\text{print} & (\text{IS\_EMPTY(Desserts)}) \\
\text{ADD} & (\text{Desserts}, 0, "cake") \\
\text{ADD} & (\text{Desserts}, 0, "pie") \\
\text{ADD} & (\text{Desserts}, 2, "cookies") \\
\text{One} & \leftarrow \text{LOOK\_UP(Desserts, 0)} \\
\text{ADD} & (\text{Desserts}, 1, "ice cream") \\
\text{Two} & \leftarrow \text{LOOK\_UP(Desserts, 2)} \\
\text{print} & (\text{One}) \\
\text{print} & (\text{Two}) \\
\text{print} & (\text{MAX\_RANKING(Desserts)})
\end{align*}
\]
Data structures for ADT Ranking

Idea 1: Array with (rank, value) pairs, unordered
Idea 2: Array with item of rank \( r \) at index \( r \)
Idea 3: Linked list, position is rank
Idea 4: Doubly-linked list, position is rank
ADT Grid

The ADT Grid allows access to data items by two indices.
Preconditions: For all \( G \) is a grid, \( \text{Num\_Rows}, \text{Num\_Cols}, \text{Row}, \) and \( \text{Col} \) are nonnegative integers, \( 0 \leq \text{Row} < \text{Num\_Rows}, 0 \leq \text{Col} < \text{Num\_Cols} \), \( \text{Data} \) any data item.
Postconditions: Mutation by \( ADD \) (adds/replaces item at \( \text{(Row, Col)} \)) and \( DELETE \) (deletes data item at \( \text{(Row, Col)} \), if any).

<table>
<thead>
<tr>
<th>Name</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CREATE(\text{Num_Rows}, \text{Num_Cols}) )</td>
<td>a new grid of dimensions ( \text{Num_Rows} \times \text{Num_Cols} ) with all entries initialized to empty</td>
</tr>
<tr>
<td>( \text{ROWS}(G) )</td>
<td>number of rows</td>
</tr>
<tr>
<td>( \text{COLUMNS}(G) )</td>
<td>number of columns</td>
</tr>
<tr>
<td>( \text{IS_EMPTY}(G) )</td>
<td>( True ) if empty, else ( False )</td>
</tr>
<tr>
<td>( \text{LOOK_UP}(G, \text{Row}, \text{Col}) )</td>
<td>data item at ( \text{(Row, Col)} ), if any, else ( False )</td>
</tr>
<tr>
<td>( \text{ADD}(G, \text{Row}, \text{Col}, \text{Data}) )</td>
<td></td>
</tr>
<tr>
<td>( \text{DELETE}(G, \text{Row}, \text{Col}) )</td>
<td>data item at ( \text{(Row, Col)} ), if any, else ( False )</td>
</tr>
</tbody>
</table>
Data structures for ADT Grid

Consider using ADT Indexed Sequence for the first dimension and then ADT Indexed Sequence again for the second dimension.

The provider for the ADT Grid is now the user for the ADT Indexed Sequence.

Key new idea
ADTs can be used in implementations.
Module summary

Topics covered:

- Case study: Racket code
- ADT Stack
- ADT Queue
- Data structure: Circular array
- Data structure: Circular linked list
- Data structure: Doubly-linked list
- Data structure: Doubly-linked circular list
- ADT Indexed Sequence
- ADT Ranking
- ADT Grid