Problem 1

Insert the numbers 12, 11, 13, 10, 20 into an empty skip list using the coin flips HHTHTHTTHHHT. Afterwards, delete 13 from the resulting skip list.

Problem 2

In this problem, we will show that deleting a single node in an AVL-tree of height \( h \) might require \( \Theta(h) \) rotations. First, we define a family \( (T_h)_{h \geq 1} \) recursively in the following manner: \( T_{-1} \) is empty and \( T_0 \) is a single node. To form \( T_h \), we start with a single node and take a copy of \( T_{h-2} \) and a copy of \( T_{h-1} \) as the left and the right children of the root, respectively.

a) For \( h \geq 0 \), what is the height of \( T_h \)? Prove your claim.

b) Prove that for \( h \geq 0 \), \( T_h \) satisfies the height requirements of an AVL tree.

c) On \( T_3 \), what are the leaves which require \( \lfloor 3/2 \rfloor = 1 \) rotation upon deletion? Pick one and show the resulting tree.

d) Same question with \( T_4 \), but now with \( \lfloor 4/2 \rfloor = 2 \) rotations.

e) Prove by induction that the above construction of \( T_h \) results in trees for which there is a node that requires \( \lfloor h/2 \rfloor \) rotations upon deletion.

Problem 3

Let \( L \) be a list of \( n \) elements. Give a sequence of \( m \) searches \((m \in \Omega(n))\) such that: (a) the average cost of a search under the MTF heuristic is \( O(1) \) and (b) the average cost of a search under the Transpose heuristic is \( O(n) \).