Key Concepts

- **Self-organizing searches** allow us to take advantage of probability distributions to access elements.
  - Optimal static ordering sorts elements by non-increasing access probability.
  - OSO is the best possible ordering, but we require complete knowledge of the probability distribution.
  - The move-to-front heuristic sorts elements by the time they were last accessed.
  - The transpose heuristic sorts elements by swapping pairs.
  - MTF is better with changing access patterns, but transpose is better with "rare" accesses.

- **A skip list** is a series of linked lists stacked on top of one another.
- Lists that form higher levels contain subsets of elements that are in lower levels.
- We traverse a skip list by skipping over elements we don’t care about and jumping down to lower levels.
- Skip lists have $O(n)$ expected space usage and $O(\log(n))$ expected height.
- Operations:
  - **Search** — $O(\log(n))$ expected, find largest key less than search key at each level and drop down
  - **Insert** — $O(\log(n))$ expected, search to determine where to insert and flip coins to determine height
  - **Delete** — $O(\log(n))$ expected, search to determine deletion positions and check for deleted key

Suggested Readings

- **Sedgewick**: 13.5 (Skip Lists)
- **Goodrich/Tamassia**: 3.5 (Skip Lists)
Practice Questions

Sedgewick

13.75. Draw the skip list that results when you insert items with the keys E A S Y Q U T I O N in that order into an initially empty list, assuming that \textsc{random} returns the sequence of values 1, 3, 1, 1, 2, 2, 1, 4, 1, and 1.

(Note: you can interpret these values as the number of heads flipped before the first tails flip.)

13.76. Draw the skip list that results when you insert items with the keys A E I N O Q S T U Y in that order into an initially empty list, assuming the same \textsc{random} return values as for exercise 13.75.

Goodrich/Tamassia

R-3.18. Draw an example skip list resulting from performing the following sequence of operations on the skip list in Figure 3.51: \textsc{remove}(38), \textsc{insert}(48), \textsc{insert}(24), \textsc{remove}(55). Assume the coin flips for the first insertion yield two heads followed by tails, and those for the second insertion yield three heads followed by tails.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{skip_list.png}
\caption{Removal of the item with key 25 from a skip list. The positions visited and the links traversed after the initial search are drawn with thick lines. The positions removed are drawn with dashed lines.}
\end{figure}

R-3.19. Give a pseudocode description of the \textsc{remove} dictionary operation, assuming the dictionary is implemented by a skip list structure.