Module 6: Tries

CS 240 - Data Structures and Data Management

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Spring 2017

Tries

- Trie (Radix Tree): A dictionary for binary strings
  - Comes from retrieval, but pronounced “try”
  - A binary tree based on bitwise comparisons
  - Similar to radix sort: use individual bits, not the whole key

- Structure of trie:
  - A left child corresponds to a 0 bit
  - A right child corresponds to a 1 bit

- Keys can have different number of bits
- Keys are not stored in the trie: a node x is flagged if the path from root to x is a binary string present in the dictionary
Tries

Example: A trie for
\[ S = \{00, 0001, 01001, 011, 01101, 01111, 110, 1101, 111\} \]

Tries: Search

\[ \text{Search}(x) : \]
- start from the root
- take the left link if the current bit in \( x \) is 0 and take the right link if it is 1; return failure if the link is missing
- if there are no extra bits in \( x \) left and the current node is flagged then - success (\( x \) is found)
- recurse
Tries: Search

Example: Search(011)
Tries: Search

Example: Search(1101)

Example of a trie for the search (1101)

Tries: Insert

- **Insert(x)**
  - Search for x, and suppose we finish at a node v
  - Note: x may have extra bits.
  - Expand the trie from the node v by adding necessary nodes that correspond to extra bits of x; flag the last one.
Tries: Insert

Example: Insert(101)

Tries: Insert

Example: Insert(0100)
Tries: Insert

Example: Insert(11101)

Tries: Delete

Delete(x)
- Search for x
- if x found at an internal flagged node, then unflag the node
- if x found at a leaf \( v_x \), delete the leaf and all ancestors of \( v_x \) until
  - we reach an ancestor that has two children or
  - we reach a flagged node
Tries: Delete

Example: Delete(011)
Tries: Delete

Example: Delete(01001)

Tries: Operations

- **Search**(x)
- **Insert**(x)
- **Delete**(x)

Time Complexity of all operations: $\Theta(|x|)$

$|x|$: length of binary string $x$, i.e., the number of bits in $x$
Compressed Tries (Patricia Tries)

- **Patricia**: Practical Algorithm To Retrieve Information Coded in Alphanumeric
- Introduced by Morrison (1968)
- Reduces storage requirement: eliminate unflagged nodes with only one child
- Every path of one-child unflagged nodes is compressed to a single edge
- Each node stores an index indicating the next bit to be tested during a search (index= 0 for the first bit, index= 1 for the second bit, etc)
- A compressed trie storing \( n \) keys always has at most \( n - 1 \) internal (non-leaf) nodes

Compressed Tries (Patricia Tries)

- Each node stores an index indicating the next bit to be tested during a search
- Example: A trie and the equivalent compressed trie
Compressed Tries: Operations

Search($x$):
- Follow the proper path from the root down in the tree to a leaf
- If search ends in an unflagged node, it is unsuccessful
- If search ends in a flagged node, we need to check if the key stored is indeed $x$

Example: Search(01001)
Compressed Tries: Operations

Example: Search(11)

Example: Search(101)
Compressed Tries: Operations

**Delete** ($x$):
- Perform Search($x$)
- if search ends in an internal node, then
  - if the node has two children, then unflag the node and delete the key
  - else delete the node and make his only child, the child of its parent
- if search ends in a leaf, then delete the leaf and
- if its parent is unflagged, then delete the parent

Example: Delete(110)
Compressed Tries: Operations

Example: Delete(011)

Example: Delete(01101)
Compressed Tries: Operations

- **Insert**($x$):
  - Perform **Search**($x$)
  - If the search ends at a leaf $L$ with key $y$, compare $x$ against $y$.
  - If $y$ is a prefix of $x$, add a child to $y$ containing $x$.
  - Else, determine the first index $i$ where they disagree and create a **new node** $N$ with index $i$.
    Insert $N$ along the path from the root to $L$ so that the parent of $N$ has index $< i$ and one child of $N$ is either $L$ or an existing node on the path from the root to $L$ that has index $> i$.
    The other child of $N$ will be a **new leaf node** containing $x$.
  - If the search ends at an internal node, we find the key corresponding to that internal node and proceed in a similar way to the previous case.

Multiway Tries

- To represent **Strings** over any **fixed alphabet** $\Sigma$
- Any node will have at most $|\Sigma|$ children
- Example: A trie holding strings \{bear, bell, ben, soul, soup\}
Multiway Tries

- Append a special **end-of-word** character, say $, to all keys
- Example: A trie holding strings \{bear, bell, be, so, soul, soup\}

![Multiway Tries Diagram]

**Compressed multi-way tries**

- Example: A compressed trie holding strings \{bear, bell, be, so, soul, soup\}