Module 6: Tries

CS 240 - Data Structures and Data Management

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Tries

- **Trie (Radix Tree):** A dictionary for binary strings
  - Comes from retrieval, but pronounced “try”
  - A binary tree based on **bitwise comparisons**
  - Similar to **radix sort**: use individual bits, not the whole key

- Structure of trie:
  - A left child corresponds to a 0 bit
  - A right child corresponds to a 1 bit

- Keys can have different number of bits

- Keys are not stored in the trie: a node \( x \) is flagged if the path from root to \( x \) is a binary string present in the dictionary

Example: A trie for \( S = \{00, 0001, 01001, 011, 01101, 01111, 110, 1101, 111\} \)
Tries: Search

Search(x):

- start from the root
- take the left link if the current bit in x is 0 and take the right link if it is 1; return failure if the link is missing
- if there are no extra bits in x left and the current node is flagged then
  - success (x is found)
- recurse

Example: Search(011)

Example: Search(0101)
Tries: Search

Example: Search(1101)

Tries: Insert

- **Insert(x)**
  - Search for x, and suppose we finish at a node v
  - Note: x may have extra bits.
  - Expand the trie from the node v by adding necessary nodes that correspond to extra bits of x; flag the last one.

Tries: Insert

Example: Insert(101)
Tries: Insert

Example: Insert(0100)

Tries: Insert

Example: Insert(11101)

Tries: Delete

Delete(\(x\))
- Search for \(x\)
- if \(x\) found at an internal flagged node, then unflag the node
- if \(x\) found at a leaf \(v_x\), delete the leaf and all ancestors of \(v_x\) until
  - we reach an ancestor that has two children or
  - we reach a flagged node
Tries: Delete

Example: Delete(011)
Tries: Operations

- **Search**($x$)
- **Insert**($x$)
- **Delete**($x$)

Time Complexity of all operations: $\Theta(|x|)$

|$x|$: length of binary string $x$, i.e., the number of bits in $x$

Compressed Tries (Patricia Tries)

- **Patricia**: Practical Algorithm To Retrieve Information Coded in Alphanumeric
- Introduced by Morrison (1968)
- Reduces **storage requirement**: eliminate unflagged nodes with only one child
- Every path of one-child unflagged nodes is compressed to a single edge
- Each node stores an **index** indicating the next bit to be tested during a search (index= 0 for the first bit, index= 1 for the second bit, etc)
- A compressed trie storing $n$ keys always has at most $n - 1$ internal (non-leaf) nodes

Compressed Tries (Patricia Tries)

- Each node stores an index indicating the next bit to be tested during a search
- Example: A trie and the equivalent compressed trie
Search($x$):
- Follow the proper path from the root down in the tree to a leaf
- If search ends in an unflagged node, it is unsuccessful
- If search ends in a flagged node, we need to check if the key stored is indeed $x$
Compressed Tries: Operations

Example: Search(101)

Delete(x):
- Perform Search(x)
- if search ends in an internal node, then
  - if the node has two children, then unflag the node and delete the key
  - else delete the node and make his only child, the child of its parent
- if search ends in a leaf, then delete the leaf and
- if its parent is unflagged, then delete the parent

Example: Delete(110)
Compressed Tries: Operations

Example: Delete(011)

```
0
0
1
1
10
0 1
0 1
0
0 , −
1 , −
2 , −
2 , −
2 , 00
0001 01001 ,3 011
1111101
01101 01111
```

Compressed Tries: Operations

Example: Delete(01101)

```
0
0
1
1
10
0 1
0 1
0
0 , −
1 , −
2 , −
2 , −
2 , 00
0001 01001 ,3
1111101
01101 01111
```

Compressed Tries: Operations

- **Insert(x):**
  - Perform Search(x)
  - If the search ends at a leaf \( L \) with key \( y \), compare \( x \) against \( y \).
  - If \( y \) is a prefix of \( x \), add a child to \( y \) containing \( x \).
  - Else, determine the first index \( i \) where they disagree and create a new node \( N \) with index \( i \).

    Insert \( N \) along the path from the root to \( L \) so that the parent of \( N \) has index \( < i \) and one child of \( N \) is either \( L \) or an existing node on the path from the root to \( L \) that has index \( > i \).

    The other child of \( N \) will be a new leaf node containing \( x \).

    If the search ends at an internal node, we find the key corresponding to that internal node and proceed in a similar way to the previous case.
Multiway Tries

- To represent **Strings** over any **fixed alphabet** \( \Sigma \)
- Any node will have at most \( |\Sigma| \) children
- Example: A trie holding strings \{ bear, bell, ben, soul, soup \}

![Multiway Tries Diagram](image1)

Multiway Tries

- Append a special **end-of-word** character, say $, to all keys
- Example: A trie holding strings \{ bear, bell, be, so, soul, soup \}

![Multiway Tries Diagram](image2)

Multiway Tries

- **Compressed** multi-way tries
- Example: A compressed trie holding strings \{ bear, bell, be, so, soul, soup \}

![Multiway Tries Diagram](image3)