1. Consider the problem of sorting an array $A = (a_1, a_2, \ldots, a_n)$ of elements with multiplicities $(m_1, m_2, \ldots, m_k)$. That is, $A$ is made up of $k$ distinct elements $(y_1, y_2, \ldots, y_k)$, where $y_i$ occurs $m_i$ times in $A$. Prove that any algorithm in the comparison model requires $\Omega(n \log n - \sum_{i=1}^{k} m_i \log m_i)$ comparisons to sort $A$.

2. We have an array $A$ of $n$ non-negative integers such that each integer is less than $k$. Give an $O(n + k)$ time preprocessing algorithm such that queries of the form “how many integers are there in $A$ that are in the range $[a, b]$?” can be answered in $O(1)$ time. Note that $a$ and $b$ are not fixed; they are parameters given to the query algorithm.

3. Suppose that we have an unsorted array $A$ of $n$ non-negative integers, where $A[i] < n^c$, for some fixed constant $c$. Describe how to sort $A$ in $O(n)$ time.