1. Consider the AVL tree shown below and perform the following operations: insert 20, delete 32 and delete 96.

2. In this problem, we will show that deleting a single node in an AVL-tree of height $h$ might require $\Theta(h)$ rotations. First, we define a family $(T_h)_{h \geq -1}$ recursively in the following manner: $T_{-1}$ is empty and $T_0$ is a single node. To form $T_h$, we start with a single node and take a copy of $T_{h-2}$ and a copy of $T_{h-1}$ as the left and the right children of the root, respectively.

   (a) For $h \geq 0$, what is the height of $T_h$? Prove your claim.

   (b) Prove that for $h \geq 0$, $T_h$ satisfies the height requirements of an AVL tree.

   (c) On $T_3$, what are the leaves which require $\lfloor 3/2 \rfloor = 1$ rotation upon deletion? Pick one and show the resulting tree.

   (d) Same question with $T_4$, but now with $\lfloor 4/2 \rfloor = 2$ rotations.

   (e) Prove by induction that the above construction of $T_h$ results in trees for which there is a node that requires $\lfloor h/2 \rfloor$ rotations upon deletion.

3. Let $L$ be a list of $n$ elements. Give an sequence of $m$ search such that: (a) the average cost of a search under the MTF heuristic is $O(1)$ and (b) the average cost of a search under the Transpose heuristic is $O(n)$.