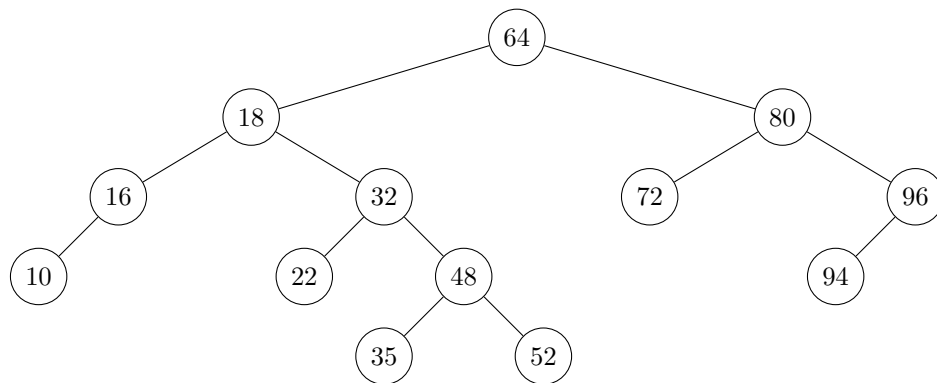


Tutorial 4: June 15

1. Consider the problem of sorting an array A of n elements each with multiplicity n/k . That is, A consists of k distinct elements (y_1, y_2, \dots, y_k) , where each y_i occurs n/k times in A . Prove that any algorithm in the comparison model requires $\Omega(n \log k)$ comparisons to sort A in the worst-case.

Note: $\forall m \geq 1, \left(\frac{m}{e}\right)^m \leq m! \leq m^m$.

2. Consider the AVL Tree shown below and perform the following operations: insert 60, delete 72, delete 48.



3. We consider a modified version of AVL trees where the height difference between the right and left subtrees of any node is in the range $[-2, 2]$ instead of $[-1, 1]$. These are called AVL-2 trees. Prove that the height of an AVL-2 tree on n nodes is in $O(\log n)$.