

Tutorial 7: July 6

1. Consider a hash table of size 7. For each of the scenarios below, insert the keys 14, 10, 20, 13, 7, 17, then delete 14 and search for 13.

a) Linear Probing with $h(k) = k \bmod 7$.

b) Double Hashing with $h_1(k) = k \bmod 7$ and $h_2(k) = (k \bmod 5) + 1$.

c) Cuckoo Hashing with $h_1(k) = k \bmod 7$ and $h_2(k) = (k \bmod 5) + 1$.

2. Suppose that we use double hashing to resolve collisions, i.e., we use the hash function $h(k, i) = (h_1(k) + ih_2(k)) \bmod m$. Show that if m and $h_2(k)$ have greatest common divisor $d \geq 1$ for some key k , then an unsuccessful insertion for key k examines $\frac{1}{d}$ th of the hash table before returning to slot $h_1(k)$.

Thus, when $d = 1$, i.e., m and $h_2(k)$ are relatively prime, then the insertion of k can only fail if every entry of the hash table is occupied.

3. Design a dictionary data structure to store key-value-pairs with uniformly distributed integer keys such that the operations for search, insert, and delete have $O(\log n)$ runtime and $O(1)$ expected runtime.