1 Assembling instructions automatically

Recall that instructions follow two basic formats, register and immediate, as detailed in the MIPS reference sheet:

Register: 0000 00ss ssst tttt dddd d000 00ff ffff
Immediate: oooo ooss ssst tttt iiiii iiiii iiiii iiiii

In both cases, ssstttttttt dddd are registers encoded as 5-bit unsigned numbers, iiiiiiiiiiiiiiiii is a two’s complement 16-bit number, and oooooo or ffffff are 6-bit opcodes specified for each instruction in their row on the sheet.

Using the bitwise operations from last week’s tutorial and the ideas from assignment 1, give pseudocode to assemble the following instructions:

1. slt $d, $s, $t.
2. beq $s, $t, i, where $i$ is an immediate value (INT or HEXINT token).

How could we design our code to maximize code reuse between various instructions?

2 Regular Languages Review

An alphabet (denoted $\Sigma$) is a finite set of symbols.

- $\{a, b, c\}$
- $\{b\}$
- $\{to, be, or, not\}$
- $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

A word (over an alphabet $\Sigma$) is finite sequence of symbols from $\Sigma$.

Example

- $bac, aba, c$ given that $\Sigma = \{a, b, c\}$
- $\epsilon, b, bb, bbb$ given that $\Sigma = \{b\}$
- to be or not to be, not to be (one word formed from the alphabet)
  $\Sigma = \{to, be, or, not\}$
DEADBEEF, FACE given that $\Sigma = \{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F\}$

A language is a set of words. A Regular Language $R$ is a set of words where either:

- $R$ is the empty language
- $R$ contains a single word
- $R$ is the union of two regular languages
- $R$ is the concatenation of two regular languages
- $R = L^* = \bigcup_{i=0}^{\infty} L^i$ where $L$ is a regular language, $L^0 = \{\varepsilon\}$ and for $i > 0$, $L^i = L \cdot L^{i-1}$

3 Regular Expressions

Regular expressions are a means of expressing regular languages using combinations of symbols and specialized operations:

- Concatenation ($ab$) - a matching word has $a$ followed by $b$
- Alternation ($a|b$) - a matching word has $a$ or $b$ but not both
- Repetition ($a^*$) - a matching word has 0 or more occurrences of $a$

Furthermore, we can group expressions into subexpressions using parenthesis. For example, $a(a|b)^*$ matches an $a$ followed by 0 or more $a$'s and $b$'s. Note that this is all essentially just shorthand for the rather verbose set notation for regular languages.

3.1 Regular Expression Problems

Build the following languages using combinations of finite languages with regular operations (set notation):

1. Construct the language of binary strings whose second letter is a '0' and whose 5th is a '1'.
2. Construct the language of binary strings that contain the substring “110101”.

Provide a regular expression for each of the following languages:

1. Convert your solutions to the two regular language problems above into regular expressions.
2. $\Sigma = \{a,b\}, L = \{aa, ab, ba, bb\}$
3. Strings over the alphabet $\Sigma = \{a,b,+,-,\cdot,\div\}$ representing valid arithmetic expressions with no parentheses. All operators should be binary (thus $a + -b$ is not valid) and multiplication must be written explicitly (thus $a \cdot b$ is valid but $ab$ is not).
4. $\Sigma = \{0, 1, 2\}, L = \{x \in \Sigma^* | x \text{ contains an even number of 0's and at least one 1.}\}$
4 Deterministic Finite Automata (DFAs)

A Deterministic Finite Automaton (DFA) is a 5-tuple \((\Sigma, Q, q_0, A, \delta)\) where:

- \(\Sigma\) – the input alphabet
- \(Q\) – finite set of states
- \(q_0 \in Q\) – a starting state in the set of states
- \(A \subseteq Q\) – set of accepting states
- \(\delta : Q \times \Sigma \rightarrow Q\) – the transition function

In the definition, \(\delta(q, \sigma)\) exists for every \(q \in Q\) and \(\sigma \in \Sigma\). However, when drawing the DFA diagram, we often assume that the transition from one state goes to error state if it was not shown in the diagram. An error state is any state that can never reach an accepting state by consuming any inputs, and error state itself can not be an accepting state.

4.1 DFA Problems

Draw DFA diagrams for the following languages:

1. The language of strings over \(\Sigma = \{a, b, c\}\) that contain exactly one a and an even number of c’s (no restriction on number of b’s).

2. The language of strings over \(\Sigma = \{a, b\}\) that contain an even number of as and an odd number of bs. How would the solution change if we added c to the alphabet and words could have any number of cs?

3. The language of strings over \(\Sigma = \{0, 1\}\) that end in 1011. How would the solution change if 1011 could appear anywhere in the string?

4. The language of strings over \(\Sigma = \{0, 1, 2, 3\}\) which are integers whose digit sum is 3. Leading zeros are permitted.

5. The language of strings over \(\Sigma = \{a, b, c\}\) that end in cab and contain an even number of a’s (no restriction on the number of b’s or c’s).