Question 1 (8 marks).
In each part, give a formula of propositional logic that corresponds to the English statement. For each atom that you use, explicitly define the English statement which it represents. Choosing which English statements to represent by atoms is part of the question.

(a) I will go to class, although I feel like staying home and playing video games.

(b) Whether or not it rains, I will go to the grocery store.

(c) If it is Winter, then both Waterloo and Toronto are cold.

(d) I will bring my umbrella, unless it is sunny.
Question 2 (9 marks).
Each of the following English statements contains a logical ambiguity. Give two formulas of propositional logic, not equivalent to one another, each of which arguably corresponds to the English statement. Explain.

(You may feel that one formula is “more correct” than the other; but you still need two.)

(a) Zhou will go to the party, unless Sam does not go to the party.

(b) $x$ is a multiple of 3 and $y$ is a multiple of 4 or $z$ is a multiple of 5.

(c) If $x$ is a multiple of 2, then $y$ is a multiple of 3, provided that $x \cdot y$ is a multiple of 6.
Question 3 (9 marks).
Given the truth table for each of the following formulas.

(a) \(((\neg(p \rightarrow q)) \rightarrow (\neg p))\). [3]

(b) \((p \rightarrow q) \rightarrow r\). [3]

(c) \((p \rightarrow (q \rightarrow r))\). [3]
Question 4 (12 marks).
Define the set of Racket expressions recursively, by

i. 2 is a Racket expression.

ii. If \( \varphi \) and \( \psi \) are Racket expressions, then
   A. \((+ \varphi \psi)\) (the sum of \( \varphi \) and \( \psi \)) is a Racket expression,
   B. \((- \varphi \psi)\) (the difference of \( \varphi \) and \( \psi \)) is a Racket expression, and
   C. \((\ast \varphi \psi)\) (the product of \( \varphi \) and \( \psi \)) is a Racket expression.

iii. Nothing else is a Racket expression.

(a) Prove by structural induction that the result of evaluating any Racket expression is an even integer (i.e. is divisible by 2).

Be careful to lay out your induction precisely.
Question 4, continued.

(b) If we add the following new axiom to the recursive definition above:

ii. D. \( \frac{\varphi}{\psi} \) (the quotient of \( \varphi \) and \( \psi \), with rounding) is a Racket expression.

then is it still true that an integer that results from evaluating any Racket expression is even? If it is true, then give a proof. If it is not true, then give a counterexample.