Due Wednesday, May 17, by noon, to Crowdmark.

All submitted work must be the student’s own.

**Question 1** (12 marks).
For each of the propositional formulas given below, determine with proof whether the formula is a contradiction (i.e. not satisfiable), satisfiable and not a tautology, or a tautology (i.e. a valid formula). Use truth tables and/or valuation trees to justify each answer.

(a) \( ((p \rightarrow q) \rightarrow (\neg p)) \)

(b) \( ((\neg(p \rightarrow q)) \rightarrow p) \)
Question 2 (12 marks).
Prove or disprove each of the following semantic entailment statements. Use truth tables and/or valuation trees to justify your answers.

(a) \{ (p \rightarrow q), ((p \rightarrow q) \rightarrow p) \} \models p  

(c) \(((p \rightarrow q) \rightarrow p) \land (\neg p))
(b) \( \{ (p \rightarrow q), ((p \rightarrow q) \rightarrow p), (r \land (\neg r)) \} \vDash (((p \rightarrow q) \rightarrow p) \land (\neg p)) \)
(c) \{(p \rightarrow q), ((p \rightarrow q) \rightarrow r)\} \models (r \rightarrow q)
**Question 3** (9 marks).
Consider the two fragments of code given below, where \( P_1, P_2, \) and \( P_3 \) are blocks of code.

**Fragment #1**

```cpp
if ( !a || b ) {
    if ( !a && !b ) { \( P_1 \) }
} else {
    \( P_2 \)
} if ( ( !a || b ) || ( a && !b ) ) { \( P_3 \) }
```

**Fragment #2**

```cpp
if ( !a && !b ) { \( P_1 \) }
else if ( !b ) { \( P_2 \) }
\( P_3 \)
```

(a) For each fragment, express in propositional logic the conditions under which each of the blocks of code \( P_1, P_2, \) and \( P_3 \) will be executed. Do NOT simplify for this part.

For Fragment #1:

- \( P_1: \)
- \( P_2: \)
- \( P_3: \)

For Fragment #2:

- \( P_1: \)
- \( P_2: \)
- \( P_3: \)

(b) Using the algebraic equivalence rules, show that Fragment #1 and Fragment #2 have the same behavior.
Question 4 (8 marks).
This problem is about adequate sets of connective symbols. Recall that a set \( S \) of connective symbols is adequate for propositional logic if every propositional formula can be written using only the connective symbols from \( S \).

(a) Let \( \downarrow \) be a connective symbol, with the value of \( p \downarrow q \) being given in the following table.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \downarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Prove that \( \{ \downarrow \} \) is an adequate set of connectives for propositional logic.

(b) Prove that \( \{ \lor, \land \} \) is not an adequate set for propositional logic.