Due Wednesday, May 24, by noon, to Crowdmark.

All submitted work must be the student’s own.

**Question 1** (6 marks).
Converting each of the following formulae to an equivalent formula in conjunctive normal form (CNF). Simplify as you go, using appropriate algebraic identities.

Show each of your steps. If the same identity applies to each of several subformulae, you may replace all of them at once.

(a) \((\neg(p \lor q)) \lor r)\).

(b) \(((\neg r) \lor (\neg p)) \land ((\neg q) \leftrightarrow r) \land ((\neg s) \leftrightarrow p) \land (\neg(q \lor s))\).
Question 2 (8 marks).
Give a Resolution refutation \( \left\{ ((\neg p) \rightarrow q) \land ((\neg p) \rightarrow r) \right\} \vdash_{\text{Res}} (p \lor (q \land r)) \), as follows.

(a) Convert the appropriate formulæ to CNF, and list the initial clauses for the resolution.

(b) Give each step of the resolution, arranged either as a list of clauses with reasons or as a tree.
Question 3 (18 marks).
The following semantic entailment statements may be either true or false. For each, do the following.

i. Using semantic arguments (truth tables, valuation trees, algebraic equivalences, etc.), determine whether or not the entailment holds.

ii. Do the corresponding Resolution proof. Explain how the proof corroborates the first argument.

(For each entailment, give each of i. and ii. separately; do not refer to work from the other part.)

\[3, 3\] 
(a) \(\{((q \lor r) \leftrightarrow s)\} \vdash (q \leftrightarrow s).\)
(b) \ \{(q \leftrightarrow r), \ ((\neg s) \leftrightarrow (\neg r))\} \models (q \leftrightarrow s).
(c) \{((\neg r) \leftrightarrow s), (q \lor (\neg r)), (s \leftrightarrow (\neg p)), (\neg (\neg p))\} \models q.
Question 4 (8 marks).

Remark on Notation: In this question, we use the associativity of $\land$ and $\lor$ to omit parentheses where doing so does not create ambiguity.

(a) Let $p_0, p_1, p_2, \ldots$ be propositional variables. Let $n$ be any natural number. Prove, using induction on $n$, the following Generalized DeMorgan Law, for any natural number $n$:

$$\neg(p_n \land \cdots \land p_0) \equiv \neg p_n \lor \cdots \lor \neg p_0.$$
(b) Let $p_0, p_1, p_2, \ldots$ be propositional variables. Let $n$ be any natural number. Let

$$\Sigma_n = \{p_0, \ldots, p_n\}.$$ 

In this question, we will prove that there exists a Resolution refutation proof to witness

$$\Sigma_n \vdash_{\text{Res}} (p_n \land \cdots \land p_0).$$

Using the result of part (a), we have that the negation of the desired conclusion formula is

$$(\neg(p_n \land \cdots \land p_0)) \equiv (\neg p_n) \lor \cdots \lor (\neg p_0).$$

Thus to complete the argument, you must supply a proof, by induction on $n$, that applying Resolution to the set of formulæ

$$\{p_0, \ldots, p_n, (\neg p_n) \lor \cdots \lor (\neg p_0)\}$$

yields $\bot$, for any natural number $n$. 