Study Exercises

To assist you with thinking about the topics of the course and about how to solve the problems, we provide the following suggestions for exercises. You may work on them on your own or with classmates (or both) as you choose. When you get stuck, please feel free to ask an instructor or IA.

How many exercises you work on is entirely up to you. We do suggest, however, that knowing the answers is NOT the important part. The important part is to practice finding such answers yourself. You will need to do this on exams.

**SE 1.** Huth & Ryan, Exercise 1.4.6, p. 85.

**SE 2.** Huth & Ryan, Exercises 1.4.7, 1.4.8, and 1.4.10, pp. 85–86.

**SE 3.** For each of the propositional formulas given below, answer all three of the following questions: Is the formula satisfiable? Is the formula a tautology? Is the formula a contradiction? Use truth tables and/or valuation trees to justify your answers.

(a) \(((p \land q) \rightarrow r) \rightarrow ((p \rightarrow r) \lor (q \rightarrow r))\)
(b) \(((\neg(p \rightarrow q)) \rightarrow ((\neg p) \rightarrow (\neg q)))\)
(c) \(((p \land (\neg r)) \land ((\neg(\neg(r \land q))) \land (r \land (\neg q)))\)

**SE 4.** Prove or disprove each of the following equivalence statements. Use proofs using logical equivalences (identities) to justify your answers. Some of these were done in class.

(a) \(((p \land q) \lor (q \lor r)) \equiv (q \land (p \lor r))\)
(b) \((p \lor (p \land q)) \equiv p\)
(c) \((\neg((\neg p) \lor (\neg(r \lor s)))) \equiv ((p \land r) \lor (p \land s))\)
(d) \((\neg((\neg(p \land q)) \lor p)) \equiv F\)
(e) \(p \equiv (p \land (q \rightarrow p))\)
(f) \(p \equiv (p \land ((\neg((\neg(q) \land (\neg p))) \lor p))\)
(g) \((p \land ((\neg((\neg(q) \land (\neg p))) \lor p)) \equiv q\)

**SE 5.** Prove or disprove each of the following semantic entailment statements. Justify your answers.

(a) \{\((\neg p) \rightarrow (\neg r)), (\neg(p \land q))\} \models (r \rightarrow q)\)
(b) \{p, (p \rightarrow q), (q \rightarrow r)\} \models r\)
(c) \{(p \rightarrow q), (q \rightarrow r), (\neg r)\} \models (\neg p)\)
(d) \{((\neg p) \rightarrow (q \lor r)), (p \rightarrow q), (\neg q)\} \models r\)
(e) \{((p \lor r) \lor (p \rightarrow q)), (\neg(p \rightarrow q)), (r \rightarrow (p \rightarrow a))\} \models p

(f) \{(p \rightarrow (q \lor r)), (p \rightarrow (\neg r)), p\} \models q

(g) \{(p \rightarrow (q \lor r)), (p \rightarrow (\neg r)), p\} \models (\neg q)

**SE 6.** Consider the two fragments of code given below, where \(P_1\), \(P_2\), \(P_3\), and \(P_4\) are blocks of code.

<table>
<thead>
<tr>
<th>Fragment #1</th>
<th>Fragment #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>if ( a \lor \neg b ) {</td>
<td>if ( a \land b ) {</td>
</tr>
<tr>
<td>\quad if ( a \land b ) { (P_1) }</td>
<td>\quad (P_1)</td>
</tr>
<tr>
<td>\quad else if ( \neg b ) { (P_2) }</td>
<td>\quad else if ( b ) {</td>
</tr>
<tr>
<td>\quad else { (P_3) }</td>
<td>\quad (P_4)</td>
</tr>
<tr>
<td>}</td>
<td>}</td>
</tr>
<tr>
<td>else {</td>
<td>else {</td>
</tr>
<tr>
<td>\quad (P_4)</td>
<td>\quad (P_2)</td>
</tr>
<tr>
<td>}</td>
<td>}</td>
</tr>
</tbody>
</table>

For each of the fragments, express in propositional logic the conditions under which each of the blocks of code \(P_1\), \(P_2\), and \(P_4\) will be executed. For any unreachable (dead) code, give an equivalence proof that the condition under which the code would be executed are a contradiction (equivalent to false). For any reachable code, give an equivalence proof that the conditions under which the code would be executed are equivalent in both fragments.