Exercise 1.
Give proofs of the following, in Propositional logic. If you find proofs that use one or more derived rules, use them as hints to find a proof using only basic rules. (Note: the laws of Boolean algebra are not proof rules. Do not use them in a formal proof.)

(a) \{ (\phi \lor \eta) \} \vdash_{ND} (\neg((\neg\phi) \land (\neg\eta))).
(b) \{ ((\neg(\phi \land \eta)) \} \vdash_{ND} ((\neg\phi) \lor (\neg\eta)).
(c) \{ ((\phi \land \eta) \lor (\phi \land \zeta)) \} \vdash_{ND} (\phi \land (\eta \lor \zeta)).
(d) For additional practice, see any of Exercises 1.2.1–3 in Huth and Ryan, pp. 78–80. (You won’t have time to do all of them!)

Exercise 2.
Prove that for any set of Propositional formulæ \( \Sigma \) and any propositional variables \( p \) and \( q \), if \( \Sigma \vdash_{ND} p \) then \( \Sigma \vdash_{ND} ((\neg p) \rightarrow q) \).

Exercise 3.
Let \( \alpha_1, \alpha_2 \) and \( \beta \) be Propositional formulæ, where \( \emptyset \vdash_{ND} (\alpha_1 \rightarrow \alpha_2) \). Let \( \Sigma \) be a set of formulæ. Show that if \( \Sigma \cup \{ \alpha_2 \} \vdash_{ND} \beta \) then also \( \Sigma \cup \{ \alpha_1 \} \vdash_{ND} \beta \).

Exercise 4.
Give proofs in Natural Deduction for each of the following. Also explain why each corresponding entailment holds.

(a) \{ ((\exists x \alpha) \lor (\exists x \beta)) \} \vdash_{ND} (\exists x (\alpha \lor \beta)).
(b) \{ ((\exists x (\alpha \lor \beta)) \} \vdash_{ND} ((\exists x \alpha) \lor (\exists x \beta)).
(c) \{ ((\exists x (\alpha \land \beta)) \} \vdash_{ND} ((\exists x \alpha) \land (\exists x \beta)).
(d) \{ ((\forall x (\alpha \land \beta)) \} \vdash_{ND} ((\forall x \alpha) \land (\forall x \beta)).
(e) \{ ((\forall x \alpha) \land (\forall x \beta)) \} \vdash_{ND} (\forall x (\alpha \land \beta)).
(f) \{ ((\forall x \alpha) \lor (\forall x \beta)) \} \vdash_{ND} (\forall x (\alpha \lor \beta)).

Exercise 5.
For additional practice, see any of Exercises 2.3.1–7,11 in Huth and Ryan, pp. 78–80. (You won’t have time to do all of them!)