Question 1 (30 marks).

[Learning Goal: Translate English sentences into compound propositions.]

Translate the following sentences into well-formed formulae of Propositional Logic. Write the truth table for your answer.

- Explicitly state which statements you will represent by which Propositional variables.
- Make sure that you define propositional variables for atomic propositions only. (An atomic proposition should not contain a negation in it, such as “I do not like Brussels sprouts.”).
- Write the well-formed Propositional formula. (No further explanation of the formula is required, but including an explanation may help the marker in some cases.)

(a) I will fly to Vancouver although I like taking the train.

(b) I will go to work only if I am not tired.

(c) I will go to the doctor despite not being sick.
(d) I will play hockey in lieu of playing soccer.

(e) Whether or not it is snowing, the University of Waterloo will be open.

(f) I do homework on every day except on Sundays.

(g) I want to buy a sedan or a hummer, but not both.
(h) My stapler does not have staples, the office does not have any staples and the manufacturer is on back-order for staples.

(i) I am feeling very sick nonetheless I wrote my exam.

(j) I am going to the beach if it is sunny.
Question 2 (9 marks).

[Learning Goals: Identify ambiguities in the English Language]

Each of the following English statements contains a logical ambiguity. Give two formulas of propositional logic, not logically equivalent to one another, each of which arguably corresponds to the English statement. You need not show a proof that your two formulations are not logically equivalent, though you might want to ensure this is the case. Explain.

(You may feel that one formula is “more correct” than the other; but you still need two.)

(a) I will walk to school if and only if my car is broken and I feel energetic

(b) I will not walk to the pharmacy or walk my dog.

(c) Gordon Ramsey has made 100 dishes but I have not had one of them.
Question 3 (10 marks).

[Learning Goals: Write a parse tree for a given formula. Prove that a well-formed propositional formula has a property using structural induction.]

(a) Give the parse tree for \(((\neg r) \rightarrow (q \land \neg p))\).
(b) Prove that every well-formed Propositional formula contains at least one propositional variable.

Be careful to lay out your induction precisely. You may group similar cases together.