Due Wednesday, September 19, by 4:00pm, to Crowdmark.

All submitted work must be the student's own.

Question 1 (30 marks).

[Learning Goal: Translate English sentences into compound propositions.]

Translate the following sentences into **well-formed** formulæ of Propositional Logic. Write the truth table for your answer.

- Explicitly state which statements you will represent by which Propositional variables.
- Make sure that you define propositional variables for atomic propositions only. (An atomic proposition should not contain a connective not a negation in it, such as "I do not like Brussels sprouts.").
- Write the well-formed Propositional formula. (No further explanation of the formula is required, but including an explanation may help the marker in some cases.)

[3] (a) I will take the red pill or the blue pill.

[3] (b) Besides going to work, I am also going to see my doctor.

[3] (c) If I play backgammon with my friend, then I will not go to the club.

[3] (d) I will play basketball instead of playing tennis.

[3] (e) Wherever there is a will, there is a way.

[3] (f) Excluding Sundays, I eat vegetables every day.

[3] (g) I want to go apple picking or carve a pumpkin, but not both.

[3] (h) If I am not sick and if I do not have debt, I will go on a cruise.

[3] (i) I will take a break if I am tired.

[3] (j) I will take a break only if I am tired.

Question 2 (9 marks).

[Learning Goals: Identify ambiguities in the English Language]
Each of the following English statements contains a logical ambiguity. Give two well-formed formulas of propositional logic, not logically equivalent to one another, each of which arguably corresponds to the English statement. You need not show a proof that your two formulations are not logically equivalent, though you might want to ensure this is the case. Explain.

(You may feel that one formula is "more correct" than the other; but you still need two.)

[3] (a) I like the rain if and only if I am working from home and I have fixed my roof.

[3] (b) I will not eat cake or eat ice cream.

[3] (c) I have a quarter if I do not have a nickel or have a dime.

Question 3 (10 marks).

[Prove that a well-formed propositional formula has a property using structural induction.]

[10] Prove that every well-formed Propositional formula contains more propositional variables than $\wedge$ connectives.

Be careful to lay out your induction precisely and explicitly define the statement you are trying to prove. You may group similar cases together.