Question 1 (20 marks).

[Learning Goal: Translate English sentences into compound propositions.]

Translate the following sentences into well-formed formulæ of Propositional Logic.

- Explicitly state which statements you will represent by which Propositional variables.
- Write the well-formed Propositional formula. (No further explanation of the formula is required, but including an explanation may help the marker in some cases.)

(a) I will go to school, although I feel like sleeping in.

(b) I will go to class, although I feel like staying home and playing video games.

(c) My exam will be easy to mark if and only if my writing is easy to read.
(d) Whether or not it rains, I will go to the grocery store.

(e) If there is a thunderstorm then I won’t go to class, but I will do my assignment.

(f) If it is raining or it is not sunny, then I carry an umbrella.

(g) I will bring my umbrella, unless it is sunny.
(h) Zhou will go to the party, unless Sam does not go to the party.

(i) If $y$ is an integer then $z$ is not real, provided that $x$ is rational.

(j) Two lines will intersect if they are not parallel.
Question 2 (8 marks).

[Learning Goals: Determine and give reasons for whether a given expression is a well-formed formula or not; Write the parse tree for a well-formed formula.]

Let \( \varphi \) be the expression
\[
p \land q \rightarrow r.
\]
Note that \( \varphi \) is not a well-formed formula of Propositional Logic.

(a) Give one way of adding parentheses to \( \varphi \) to produce a well-formed formula of Propositional Logic.

(b) Give the parse tree for well-formed formula of Propositional Logic which you produced in part 2a.

(c) Give a different way from the choice you made in part 2a of adding parentheses to \( \varphi \) to produce a well-formed formula of Propositional Logic.

(d) Give the parse tree for well-formed formula of Propositional Logic which you produced in part 2c.
Question 3 (12 marks).

[Learning Goals: Write the parse tree for a well-formed formula; Prove properties of a recursively defined concept using structural induction.]

The Polish notation for formulas is defined as follows.

- Any propositional variable is a formula in Polish notation (an atom).
- If $\alpha$ is a formula in Polish notation, then $\neg \alpha$ is a formula in Polish notation.
- If $\alpha$ and $\beta$ are formulæ in Polish notation, then $\land \alpha \beta$ is a formula in Polish notation.
- Nothing else is a formula in Polish notation.

Throughout this question, let $\gamma$ be the formula $\land \neg p \land \neg q \land \neg r$ in Polish notation.

(a) Give the parse tree for $\gamma$.

(b) Give the well-formed formula of Propositional Logic that corresponds with $\gamma$. 
(c) Let \( \varphi \) be any formula in Polish notation. Prove by structural induction on \( \varphi \) that \( |\varphi| \geq 1 \).

Recall that \( |\varphi| \) denotes the length of \( \varphi \), i.e. the number of symbols in \( \varphi \).

Be careful to lay out your induction precisely.