Due Wednesday, September 27, by 4pm, to Crowdmark.
All submitted work must be the student’s own.

Question 1 (12 marks).

Learning goals: Define a (truth) valuation; Determine the truth value of a formula by using a truth table; Determine the truth value of a formula by using a valuation tree.

Consider the propositional formula \(((a \rightarrow b) \land (c \lor \neg b)) \lor (c \leftrightarrow a)\).

(a) (4 marks) Write the truth table for the formula. Make sure you provide intermediate steps for \((a \rightarrow b), (c \lor \neg b),\) and \((c \leftrightarrow a)\).

(b) (4 marks) Write a valuation tree for the formula.

(c) (2 marks) Give a truth valuation and show that the formula is true under this truth valuation. Please justify your answer by referring to the corresponding row in the truth table or the corresponding leaf node in the valuation tree.

(d) (2 marks) Give a truth valuation and show that the formula is false under this truth valuation. Please justify your answer by referring to the corresponding row in the truth table or the corresponding leaf node in the valuation tree.
Question 2 (18 marks).

Learning goals: Define tautology, contradiction, and satisfiable formula; Prove whether a formula is a tautology, a contradiction, or satisfiable, using a truth table and/or a valuation tree.

For each of the propositional formulas given below, determine whether the formula is a tautology, whether it is a contradiction, and whether it is satisfiable but not a tautology. Justify each answer by using a truth table, or by using a valuation tree, or by giving a truth valuation, or by writing down your reasoning in English.

(a) (6 marks) \(((a \land b) \rightarrow b)\)

(b) (6 marks) \(\neg((c \land b) \rightarrow ((\neg a) \lor c))\)

(c) (6 marks) \(((p \land q) \rightarrow (\neg r)) \land (q \lor p)\)
Question 3 (15 marks).

Learning goal: Prove the logical equivalence of formulas using logical identities.

In lectures, we saw that we could prove the equivalence of two pieces of code fragments using logical identities. In this question, we look at a different application of logical identities: using it to prove the equivalence of digital circuits.

The professor assigned a circuit design problem and three students came up with three different solutions, as shown below.

![Figure 1: Circuit 1](image1)

![Figure 2: Circuit 2](image2)

![Figure 3: Circuit 3](image3)

The students claim that all three circuits are equivalent but the marker isn’t convinced. Please help the marker by proving that these three circuits are equivalent to one another. For your convenience, we have translated three circuits into propositional logic formulas for you:

1. \(((p \land (\neg q)) \lor ((\neg p) \land q)) \lor (\neg (p \lor q)))\)
2. \(((p \lor q) \land (\neg (p \land q))) \lor (\neg (p \lor q)))\)
3. \((\neg p) \lor (\neg q)\)

This question continues on the next page.
Prove the equivalence of the three circuits by answering the two questions below.

(a) (9 marks) Prove that circuits 1 and 3 are logically equivalent using logical identities. Do not use a truth table or a valuation tree.

(b) (6 marks) Prove that circuits 2 and 3 are logically equivalent using logical identities. Do not use a truth table or a valuation tree.
**Question 4** (8 marks).

*Learning goal: Prove that a set of connectives is an adequate set for propositional logic by using truth tables and logical identities; Prove that a set of connectives is not an adequate set for propositional logic.*

This problem is about **adequate sets of connective symbols**. Recall that a set $S$ of connective symbols is **adequate for propositional logic** if every propositional formula can be written using only the connective symbols from $S$.

(a) (4 marks) Let ‘↓’ be a connective symbol, with the value of $(p \downarrow q)$ being given in the following table.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$(p \downarrow q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
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<td>F</td>
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<td>F</td>
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<td>T</td>
</tr>
</tbody>
</table>

Prove that $\{\downarrow\}$ is an adequate set of connectives for propositional logic.

(b) (4 marks) Prove that $\{\rightarrow, \lor\}$ is **not** an adequate set for propositional logic.