Due Friday, Oct.12, by 4pm, to Crowdmark.

All submitted work must be the student's own.

Question 1 (12 marks).
Prove each of the following using Natural Deduction. For full marks, restrict your proof to only the basic rules. At most one point will be awarded for a correct proof using derived rules.

Remember to number all of your lines and justify each line in your proof.

[2] (a) $\{p\} \vdash_{ND} (q \to p)$

[2] (b) $\{(p \vee q), (\neg p)\} \vdash_{ND} q$

[2] (c) $\{(p \wedge (q \vee r))\} \vdash_{ND} ((p \wedge q) \vee (p \wedge r))$

[3] (d) $\{(p \wedge q)\} \vdash_{ND} \big(\neg((\neg p) \vee (\neg q))\big)$

[3] (e) $\emptyset \vdash_{ND} (((p \to q) \to p) \to p)$

Question 2 (14 marks).

For this question, consider the following premises:

> *"Logic is difficult or it is not fun."*
> *"If computer science is easy, then logic is not difficult."*
> *"CS 245 is not easy only if it is not the case that both computer science is easy and logic is fun."*
> *"Computer science is not easy if logic is not fun."*
> *"If CS 245 is easy then logic is fun."*

[2] (a) For each premise, translate into propositional well-formed formulas. Precisely state what each propositional variable means and keep each propositional variable as the "positive" form of the proposition. Do not simplify your formulas.

[12] (b) For each of the following parts, translate the given statement into propositional well-formed formulas and determine whether or not the statement is derivable from the set of premises. If it is derivable, give the Natural Deduction proof from the set of premises to the statement. Else, prove that no such proof exists in Natural Deduction.
(You may use modus tollens, law of excluded middle, proof by contradiction, or double negation introduction without loss of credit).

[4] i. "Computer science is easy or logic is difficult."

[4] ii. "Computer science is not easy."

[4] iii. "If CS245 is easy, then logic is not difficult."

Question 3 (6 marks).
Let α and β be propositional well-formed formulas.
For the following, determine whether or not the given inference rule is sound.

Note: To show that an inference rule is sound, show that the inferred formula is semantically entailed from the given formulas in the rule. To show that it is not sound, provide a counter-example (define your formulas and truth valuation) where the entailment does not hold.

[3] (a)

$$\frac{(\alpha \to (\neg\beta)) \quad \alpha}{(\alpha \vee \beta)} \quad .$$

[3] (b)

$$\frac{(\alpha \to \beta) \quad (\beta \to \alpha)}{(\alpha \wedge \beta)} \quad .$$

Question 4 (6 marks).
Let Σ be a set of propositional well-formed formulas.
Let α be a propositional well-formed formula.

Without using Soundness or Completeness, prove the following statement:

$$\Sigma \cup \{(\neg\alpha)\} \vdash_{ND} \perp \quad \text{if and only if} \quad \Sigma \vdash_{ND} \alpha$$

Hint: Use the Natural Deduction proofs.