Question 1 (12 marks).
Show that each of the following Hoare triples is satisfied under partial correctness where the domain
is the natural numbers (including 0). Give all the needed details of your program annotation. For
“implied” proofs, use ordinary arithmetic laws.

(a) \(\{ \text{true} \} \)

\( y = x \)
\( x = x + 1 \)
\( z = x \times y \)

\(\{ \exists u ((u + u) = z) \} \)
(b) \{ x = x_0 \}
    if (x < 0) {
        x = -x;
    }
\{ ((x_0 < 0 \land x = -x_0) \lor (x_0 \geq 0 \land x = x_0)) \}
(c) \{ true \}
    if ( y <= z ) {
        x = z - y ;
    } else {
        x = y - z ;
    }
\{ (x \geq 0) \land (((x + y) = z) \lor (y = (x + z))) \}
Question 2 (8 marks).
Consider the following specification:
\[ y = y_0 \land y \geq 0 \]
z = 0 ;
while ( y > 0 ) {
    z = z + x ;
    y = y - 1 ;
}
\[ z = x \cdot y_0 \]

(a) Prove that this specification is satisfied under partial correctness. Show all the needed details of your program annotation, and explicitly prove any implications required by your annotation.
(b) Prove that this specification is satisfied under total correctness. You may assume the partial correctness from part (a), whether or not you solved it.
**Question 3** (26 marks).

Consider the following specification where the domain is the integers:

\[ (a > 0 \land b_0 = b) \]

if \( b < 0 \) {
    \begin{align*}
    & y = 0 ; \\
    & \text{while} \ ( b < 0 ) \ { \\
    & \quad y = y - a ; \\
    & \quad b = b + 1 ; \\
    & \}
    \}

} else {
    \begin{align*}
    & y = 1 ; \\
    & \text{while} \ ( b > 0 ) \ { \\
    & \quad y = y \times a ; \\
    & \quad b = b - 1 ; \\
    & \}
    \}

\[ (b_0 < 0 \land y = a \cdot b_0) \lor (b_0 \geq 0 \land y = a^{b_0}) \]

(a) Show all the needed details of the program annotation which is required to prove that the given specification is satisfied under partial correctness.
(b) Justify any implications required by your annotation from part (a). Your justifications for your required implications need NOT be formal natural deduction proofs.
(c) Prove that this specification is satisfied under total correctness. You may assume the partial correctness from parts (a) and (b), whether or not you solved them.