Due Wednesday, Jul. 11, by 4:00pm, to Crowdmark.

All submitted work must be the student’s own.

Question 1 (11 marks).

[Learning Goals: Translate English sentences into propositions. Prove a conclusion from given premises using Natural Deduction inference rules.]

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

A newcomer came to the island and talked to every person on the island. Every person on the island told the newcomer that “All of us on the island are of the same type.”

Using the power of logic, the newcomer concluded that every person on the island is a knight.

(a) Translate the argument into Predicate formulas. Clearly indicate which formulas are the premises and which formula is the conclusion.

This part of the question is worth 0 marks. We will provide the solution to you on the next page. However, you are strongly encouraged to attempt this question before looking at the solutions.
Solutions to part a:

Define \( P(x) \) to mean that that person \( x \) is a knight. We have translated the above argument into the following Predicate formulas.

Premise:

\[
(\forall x ((P(x) \rightarrow ((\forall y P(y)) \lor (\forall z (\neg P(z)))) \land ((\forall y P(y)) \lor (\forall z (\neg P(z)))) \rightarrow P(x)))
\]

In the following parts, let \( \alpha \) denote the premise.

Conclusion: \((\forall x P(x))\)

In the following parts, you will construct a Natural Deduction proof for the above argument.

You may use any of the derived rules MT (modus tollens), PBC (proof by contradiction), LEM (law of excluded middle), or \( \neg\neg \), without loss of credit.

You may use any parts of question 1 and 2 from assignment 7 even if you did not complete the proofs.

This Natural Deduction proof is fairly complicated. Please complete the three shorter proofs in parts b, c, and d below. In part e, you will put these proofs together to form the final proof.

(b) If islander \( x_0 \) is a knave, then ((not every islander is a knight) and (not every islander is a knave)).

\[
\{\alpha, (\neg P(x_0))\} \vdash ((\neg (\forall y P(y))) \land (\neg (\forall z (\neg P(z)))).
\]
(c) If islander $x_0$ is a knave, then there exists a knight on the island.

$$\{\alpha, (\neg P(x_0))\} \vdash (\exists z (\neg (\neg P(z)))).$$  

(d) If islander $z_0$ is a knight, then ((every islander is a knight) or (every islander is a knave)).

$$\{\alpha, P(z_0)\} \vdash ((\forall y P(y)) \lor (\forall z (\neg P(z)))).$$
(e) Provide a Natural Deduction proof for the entire argument.

\{\alpha\} \vdash (\forall x \ P(x)).
Question 2 (6 marks).

[Learning Goals: Translate English sentences into Predicate Logic formulæ. Answer questions based on the formulæ]

Let $\{P\} \ C \ \{Q\}$ be a Hoare triple. Consider the following:

Domain

"all possible program states".

Function Symbols with their interpretations

$f(s)$: returns the state $t$ after running code $C$ starting with state $s$.

Predicate Symbols with their interpretations

$P(s)$: “state $s$ satisfies precondition $P$”.
$Q(s)$: “state $s$ satisfies postcondition $Q$”.
$C(s, t)$: “code $C$ begins in state $s$ and terminates in state $t$”.

Logical Connectives

$\rightarrow, \neg, \lor, \land$

Quantifiers

$\forall, \exists$

Using only the above, translate the following sentences into well-formed formulæ of Predicate logic. Please ensure your translations follow from the given sentences.

(a) Write the definition of partial correctness in Predicate logic. That is, write

for every state $s$ that satisfies condition $P$, if execution of $C$ starting from state $s$ terminates in a state $s'$, then state $s'$ satisfies condition $Q$.

using Predicate logic.

(b) Write the definition of total correctness in Predicate logic. That is, write

for every state $s$ that satisfies $P$, execution of $C$ starting from state $s$ terminates in a state $s'$ and the resulting state $s'$ satisfies $Q$.

using Predicate logic. You may use your answer from 2a as a subformula.
Question 3 (16 marks).
Show that each of the following Hoare triples is satisfied under partial correctness where the domain for variable values is the integers. Give all the needed details of your program annotations. Explicitly prove any implications required by your annotations. Assume all symbols below carry their usual arithmetic meanings.

(a) ⦇true⦈
    y = x ;
    x = x + 2 ;
    z = y * x * (x-1) ;
    ⦇∃u ((u + (u + u)) = z)⦈
(b) \textit{true}

\begin{verbatim}
if (x * y < 0)
    z = x * (x - y);
else
    z = x * (x + y);
\end{verbatim}

\begin{verbatim}
\(((((x < 0) \land (y > 0)) \lor ((x > 0) \land (y < 0))) \land (z = ((x \cdot x) - (x \cdot y))))
\end{verbatim}

\begin{verbatim}
\lor ((((x \geq 0) \land (y \geq 0)) \lor ((x \leq 0) \land (y \leq 0))) \land (z = ((x \cdot x) + (x \cdot y))))\end{verbatim}
(c) \((x = x_0)\) \\
\text{if} (x > 0) \{ \\
\quad x = -x; \\
\} \\
\text{\((x = -|x_0|)\)}
(d) \(\text{true}\)
\[
z = (a \times a \times a \times a \times b) \mod 5;
\]
\[
\text{if } ((z \mod 5) \neq 0) \{
\quad a = b;
\}
\]
\(\langle a \equiv b \mod 5 \rangle\)

(A hint about what formula can be placed before the if statement can be found on Piazza if needed).
Question 4 (5 marks).

[Learning Goals: Research and write a report on a bug that occurred and describe the details and ramifications to the company] In class, many of us mentioned some of the many bugs that occurred that have negatively impacted companies. These include

- 2018 - Spectre and Meltdown flaw with Intel’s chipset causing computers to potentially reveal restricted information
- 2012 - Knight Capital Group high-frequency trading system lost $440 million dollars in 30 minutes.
- 2007 - Toyota Prius recalled 160,000 vehicles due to unexpected stalling
- 2003 - Power black-out caused 50 million people in North America to be without power.

and many others. In this question, we would like you to research one of these bugs above (or choose one that you are interested in), explain what happened and how the bug was caused. Marks will be given on quality of references used, description of the bug and overall content, spelling and grammar. Your report/paragraph should not exceed 200 words. Ensure you are citing your references properly. See


http://subjectguides.uwaterloo.ca/c.php?g=695489&p=4932734

for some details on how to cite references (for example, make sure that webpages used have the date accessed!).