Study Exercises

Exercise 1. [English and Propositional Logic]
Huth & Ryan, Exercise 1.1.1, p. 78. Any or all of the parts.

Exercise 2. Briefly explain why each of the following cannot be translated into Propositional Logic.
(a) Hand your solutions in to the drop box.
(b) Can this sentence be translated into propositional logic?
(c) Fruit flies like a banana.

Exercise 3. [WFFs and parse trees]
Huth & Ryan, Exercises 1.3, pp. 81–2. Any or all of the questions.

Exercise 4. [induction]
Huth & Ryan, Exercises 1.4.7–10, pp. 85–6. Any or all of the parts.

Exercise 5. The following claim is false. Thus its “proof” must contain at least one error.

**Claim:** Every non-negative integer is even.

**Proof** by mathematical induction.

**Base case:** 0 is even.

**Inductive step:** Suppose that the claim holds up to $k$.

To prove that $k + 1$ is even, note that $k + 1 = (k - 1) + 2$.

Since $k - 1 \leq k$, the hypothesis of the inductive step applies: $k - 1$ is even.

By the definition of even, $k - 1 = 2m$ for some $m \in \mathbb{Z}$.

Thus $k + 1 = (k - 1) + 2 = 2m + 2 = 2(m + 1)$, where $m + 1 \in \mathbb{Z}$.

Therefore, by the definition of even, $k + 1$ is even.

Thus, by the principle of mathematical induction, every non-negative integer is even.

(a) Specify the exact property being used in the induction. Give both the precise assumption in the inductive step and the precise conclusion of the inductive step.

(b) Explain precisely where and how the “proof” makes its mistake(s). (Do not simply say it can’t be right; identify exactly what is wrong.)
Exercise 6. The **Polish notation** for formulas is defined as follows.

- Any propositional variable is a formula in Polish notation (an atom).
- If $\alpha$ is a formula in Polish notation, then $\neg \alpha$ is a formula in Polish notation.
- If $\alpha$ and $\beta$ are formulæ in Polish notation, then $\land \alpha \beta$ is a formula in Polish notation.
- Nothing else is a formula in Polish notation.

As you will prove in this exercise, formulæ in Polish notation are unambiguous, even though they have no parentheses.

(a) For each of the following formulas in Polish notation, give the parse tree to construct it. Also give the corresponding well-formed formula of Propositional Logic.

i. $\land p \land q \neg p$.

ii. $\land \neg \land p qp$.

(b) Prove, using induction on the length of a formula, that no formula in Polish notation is a proper prefix of another formula in Polish notation. That is, if $\varphi$ is a formula in Polish notation, and $\eta$ is any expression of length $\geq 1$, then their concatenation $\varphi \eta$ is **not** a formula in Polish notation.

(c) Using the previous result, prove that every formula in Polish notation is a formula in Polish notation in only one way.