Study Exercises

The following are on pages 299–301 of the text.

Exercise 1.
Huth & Ryan’s Exercise 4.2.2 (p. 299) describes a for-statement like that found in C, C++ and related languages.

(a) Write a program equivalent to
\[
\begin{array}{c}
\text{for ( } C_1 ; B ; C_2 \text{ ) }
\end{array}
\]
which uses only while and if-then-else as control statements.

(b) Based on your equivalent code (or by direct arguments), give a deduction rule appropriate for the partial correctness of for-statements, of the following form.

\[
\frac{\text{? ? ? ?}}{\langle P \rangle \text{ for } (C_1 ; B ; C_2 ) \ C_3 \ \langle Q \rangle } \ (\text{partial-for})
\]

You will need to refer to a loop invariant.

Exercise 2.
Prove that the following Hoare triple is satisfied under partial correctness, and total correctness. Clearly state both of your loop invariants!

\[
\begin{array}{c}
\langle ((n \geq 2) \land (\forall x (((1 \leq x) \land (x \leq n)) \rightarrow (A[x] = 0))) \rangle \\
A[1] = 1; \\
i = 2; \\
\text{while (i <= n) }
\end{array}
\]

\[
\begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
i = i + 1; \\
\text{ } \\
\text{ } \\
\text{ } \\
} \\
\langle A[n] = 2^{n-2} \rangle \\
\end{array}
\]

Exercise 3.
An advanced question, for those wishing to go further.

Show that the triple

\[
\begin{array}{c}
\langle \text{true} \rangle \\
A[A[2]] = 3 \\
\langle A[A[2]] = 3 \rangle
\end{array}
\]
does NOT hold under partial correctness.

What is the proper pre-condition for the above code, to satisfy the post-condition? Does a nested array assignment (like the above) obey the array-assignment rule, or is an augmented rule required?