Due Wednesday, Oct. 3, by 4:00 pm, to Crowdmark.

All submitted work must be the student's own.

Question 1 (6 marks).
Convert each of the following formulæ to an equivalent formula in conjunctive normal form (CNF). Simplify as you go, using appropriate algebraic identities.

Show each of your steps. If the same identity applies to each of several subformulæ, you may replace all of them at once.

[3] (a) $\big((\neg(p \vee q)) \vee r\big)$.

[3] (b) $\Big(((\neg r) \vee (\neg p)) \wedge \Big((((\neg q) \leftrightarrow r) \wedge ((\neg s) \leftrightarrow p)) \wedge (\neg(q \vee s))\Big)\Big)$.

Question 2 (8 marks).
For this question, we augment the set of connective symbols to include '$\perp$' as a nullary connective. ("Nullary" means that it doesn't actually connect anything. Thus, for example, "$\perp$", "$(p \wedge \perp)$", etc., are well-formed formulas.)

[2] (a) What changes need to be made to the definition of "well-formed formula", in order to allow use of $\perp$?

[6] (b) Define that the formula $\perp$ is a contradiction; that is, $\perp^t = \mathtt{F}$, for any valuation t.

Show that the set $\{\to, \perp\}$ is an adequate set of connectives.

Question 3 (18 marks).
The following semantic entailment statements may be either true or false. For each, do the following.

i. Using semantic arguments (truth tables, valuation trees, algebraic equivalences, etc.), determine whether or not the entailment holds.

ii. Do the corresponding Resolution proof.

(For each entailment, give each of i. and ii. separately; do not rely on work from the other part.)

[3, 3] (a) $\{((q \vee r) \leftrightarrow s)\} \vDash (q \leftrightarrow s)$.

[3, 3] (b) $\{(q \leftrightarrow r),\ \big((\neg s) \leftrightarrow (\neg r)\big)\} \vDash (q \leftrightarrow s)$.

[3, 3] (c) $\{((\neg r) \leftrightarrow s),\ (q \vee (\neg r)),\ (s \leftrightarrow (\neg p)),\ (\neg(\neg p))\} \models q$.

Question 4 (6 marks).

Remark on Notation: *In this question, we use the associativity of $\wedge$ and $\vee$ to omit parentheses where doing so does not create ambiguity.*

[6] Let $p_0, p_1, p_2, ...$ be propositional variables. Let n be any natural number. Prove, using induction on n, the following **Generalized DeMorgan Law**, for any natural number n:

$$\big(\neg(p_n \wedge \cdots \wedge p_0)\big) \equiv \big((\neg p_n) \vee \cdots \vee (\neg p_0)\big) \ .$$