Propositional Logic - Exercises
Translation English – Propositional Logic

(a) Which of the following sentences are propositions? What are the truth values of those that are propositions?

1. Waterloo is the capital of Ontario.
2. Montreal is the capital of Canada.
3. $2 + 3 = 5$.
4. $5 + 7 = 10$.
5. $x + 2 = 11$.
6. Answer this question.
7. $x + y = y + x$ for every pair of real numbers $x$ and $y$.
8. Do not pass go.
9. What time is it?

(b) What is the negation of each of the following propositions?

1. Today is Thursday.
2. $2 + 1 = 3$.
3. The summer in Waterloo is hot and sunny.
Let \( p \) and \( q \) be propositions
\[ p: \text{“It is below freezing.”} \]
\[ q: \text{“It is snowing.”} \]
Write the following propositions using \( p \) and \( q \) and logical connectives.

1. It is below freezing and snowing.
2. It is below freezing but not snowing.
3. It is not below freezing and it is not snowing.
4. It is either snowing or below freezing (or both).
5. If it is below freezing, it is also snowing.
6. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
7. That it is below freezing is necessary and sufficient for it to be snowing.
For each of the following sentences state what the sentence means if the “or” is inclusive (disjunction) versus an exclusive “or”. Which of these meaning of “or” do you think is intended?

1. To take discrete mathematics, you must have taken calculus or a course in computer science.

2. When you buy a new car from Acme Motor company, you get a 2000 dollars back in cash or a 2 percent car loan.

3. Dinner for two includes two items from column A or three items from column B.

4. School is closed if more than 2 feet of snow falls or if the wind chill is below -100.
Write each of the following statements in the form “$p \rightarrow q$”.

1. It snows whenever the wind blows from the northeast.
2. The apple trees will bloom if it stays warm for a week.
3. That the Pistons win the championship implies that they beat the Lakers.
4. It is necessary to walk 8 miles to get to the top of Long’s peak.
5. To get tenure as a professor, it is sufficient to be world-famous.
6. If you drive more than 400 miles, you will need to buy gasoline.
7. Your guarantee is good only if you bought your CD player less than 90 days ago.
**Definition.** Given the proposition $p \rightarrow q$,

- the proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$
- the **contrapositive** of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

**Exercises:**
State the converse and the contrapositive of each of the following implications:

1. If it snows today, I will ski tomorrow.
2. I come to class whenever there is going to be a quiz.
3. A positive integer is a prime only if it has no divisors other than 1 and itself.
Propositional calculus: More exercises

(a) Prepare the truth tables for the exclusive or.
(b) Give the truth tables for $p \land p$, $p \lor p$, $p \land 1$, and $p \land 0$.
(c) Translate the following propositions into formulas:

1. He is clever and diligent.
2. He is clever but not diligent.
3. He didn’t write the letter, or the letter was lost.
4. He must study hard, otherwise he will fail.
5. He will fail, unless he studies hard.
6. He will go home, unless it rains.
7. He will go home only if it rains.
8. If it rains, he will be at home; otherwise, he will go to the market or school.
9. The sum of two numbers is even if both numbers are even or both numbers are odd.
Another knight/knave puzzle

There is an island in which certain inhabitants, called knights always tell the truth, and others, called knaves always lie. It is assumed that every inhabitant of this island is either a knight or a knave.

- A says: “If I’m a knight, then $2 + 2 = 4$.”

Is A a knight or a knave?