Exercises are for your own use, as you see fit. Study on your own or with friends, ask questions in office hours and/or the Tutorial Centre, etc. Do as much or as little as you feel will be helpful.

**Exercise 1.**
From the posted notes on recursion:

(a) Exercise 1.5.
(b) Exercise 1.6.
(c) Exercise 2.1.

**Exercise 2.**
(a) The quotient of $x$ by $y$ is the largest $z$ such that $y \times x \leq x$, often denoted $\lfloor x/y \rfloor$. Give a definition of quotient as a primitive-recursive function. (Dowek’s version of this question (Prop. 3.2) allows minimization; this version does not.)

Do the same for remainder (a.k.a. modulus).

(b) Show that $\text{blen}(n)$, the length of the binary representation of $n$, is primitive recursive.

(For $n \geq 1$, $\text{blen}(n) = 1 + \lfloor \log_2 n \rfloor = \lceil \log_2 (n + 1) \rceil$, but that probably doesn’t help here.)

**Exercise 3.**
(a) (Dowek Proposition 3.7.) Explain why $\text{hd}(n) < n$ and $\text{tl}(n) < n$, for every $n$.

(b) Justify that the functions ‘;’, ‘hd’ and ‘tl’ are primitive recursive. (Dowek’s Proposition 3.6, without the option of using minimization.)