ASSIGNMENT 10

DUE: Wednesday November 29, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read http://www.student.cs.uwaterloo.ca/~cs341 for general instructions and policies. In particular, note that “giving” an algorithm includes justifying correctness and run time.

For this assignment, you may assume that the following problems are NP-complete: 3-SAT, Independent Set, Clique, Vertex Cover, Hamiltonian cycle (directed and undirected), Hamiltonian path, Subset Sum.

1. [10 marks] Prove that the following problems are NP-complete:

   (a) [5 marks] Given two graphs, \( H = (V_H, E_H) \), and \( G = (V_G, E_G) \), is \( H \) a subgraph of \( G \), i.e. is there a mapping \( \pi \) of the vertices of \( H \) to the vertices of \( G \) such that \( \pi \) is one-to-one (it never maps two vertices of \( H \) to the same vertex of \( G \)) and such that for every pair of vertices \( u, v \in V_H \), we have \((u, v) \in E_H \) iff \((\pi(u), \pi(v)) \in E_G \).

   (b) [5 marks] Prove that the following problem is NP-complete. Given an edge-weighted undirected graph \( G = (V, E) \) and a number \( k \in \mathbb{N} \), does \( G \) have a simple path of length \( \geq k \)? Recall that a “simple” path is one that does not repeat vertices. Hint: even the version where all edge weights are 1 is NP-complete.

2. [10 marks] Prove that the following problem is NP-complete. Given a list of \( n \) positive integers, \( a_1, a_2, \ldots, a_n \), indexed by \( S = \{1, \ldots, n\} \), is there a partition \( S = A \cup B \) with \( A \cap B = \emptyset \) such that \( \sum_{i \in A} a_i = \sum_{i \in B} a_i \).

3. [10 marks] Prove that the following problem is NP-complete. Given a directed graph \( G \) and a number \( k \), can we remove \( \leq k \) edges to get a directed acyclic graph?

   Hints. Reduce from Vertex Cover. Suppose we have an input \( G = (V, E), k \in \mathbb{N} \), for vertex cover. Suppose graph \( G \) has \( m \) edges are \( n \) vertices \( v_1, v_2, \ldots, v_n \). Construct a directed graph \( G' \) with \( 2n \) vertices \( v_1, \ldots, v_n, v'_1, \ldots, v'_n \) and the following \( n + 2m \) edges:

   - \((v_i, v'_i) \) for all \( i = 1, \ldots, n \)
   - \((v'_i, v_j) \) and \((v'_j, v_i) \) for all \( (v_i, v_j) \in E \).