ASSIGNMENT 1

DUE: Wednesday September 18, 5 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.
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Note: All logarithms are base 2 (i.e., log \( x \) is defined as log_2 \( x \)).

1. [12 marks] Order notation. For each of the following pairs of functions \( f(n) \) and \( g(n) \)
defined on positive integers, determine the “most appropriate” symbol in the set \( \{ O, o, \Theta \} \) to
fill in the blank in the statement \( f(n) \in \_\_\_ (g(n)) \), if one of these symbols applies. “Most
appropriate” means that you should not answer “O” if you could answer “o” or “\( \Theta \)”. Justify
your answers.
You may use the following facts (based on Skiena, p. 56), where \( f(n) \ll g(n) \) is shorthand
for \( f(n) \in o(g(n)) \):

\[
1 \ll \log \log n \ll \log n \ll \sqrt{n} \ll n \ll n \log n \ll n^2 \ll 2^n \ll n!
\]
Furthermore, \( n^a \in o(n^b) \) for \( 0 < a < b \), and log^a n \in o(n^b) \) for any \( b > 0 \), and \( n^a \in o(2^n) \).

(a) \( f(n) = 123n^3 + 4000n^2 \), \( g(n) = .0001n^3 - 4000n^2 \);
(b) \( f(n) = \log \log(2n^2) \), \( g(n) = 16\log\sqrt{n} \);
(c) \( f(n) = \sqrt{n} \log n \), \( g(n) = n\sqrt{\log n} \);
(d) \( f(n) = n^3 \), \( g(n) = (\lceil \frac{n}{2} \rceil - \frac{n}{2})n^3 \);

2. [10 marks] Reductions. In class we saw an \( O(n^2) \) time algorithm for the following 3Sum
problem:

3Sum: Given an array \( A[1..n] \) of \( n \) numbers, find, if they exist, indices, \( i, j, k, 1 \leq i, j, k \leq n \)
Consider the more general problem of 3Sum with 3 different arrays:

Gen3Sum: Given three arrays \( X, Y, Z \) where \( X \) has \( n_x \) integers, \( Y \) has \( n_y \) integers, \( Z \) has
\( n_z \) integers, and \( n_x + n_y + n_z = n \), find, if they exist, indices \( i, j, k \) in the appropriate ranges
such that \( X[i] + Y[j] + Z[k] = 0 \).

(a) [2 marks] Suppose you define array \( A[1..n] \) to be the concatenation of \( X, Y \) and \( Z \) and
run the old 3Sum algorithm on \( A \). Give an example to show that does not solve the
Gen3Sum problem.
(b) [8 marks] Give a correct reduction from Gen3Sum to 3Sum. You should not give a stand-
alone algorithm or alter the 3Sum algorithm. Argue that your reduction is correct and
give the run time.

\textit{Hint.} Consider 3 arrays \( X'[i] = 10 \ast X[i] + 1 \), \( Y'[i] = 10 \ast Y[i] + 2 \), and \( Z'[i] \) defined in
some similar manner that you should figure out.